College Algebra — Final Exam
MAT 140, Fall 2012 — D. Ivanšić

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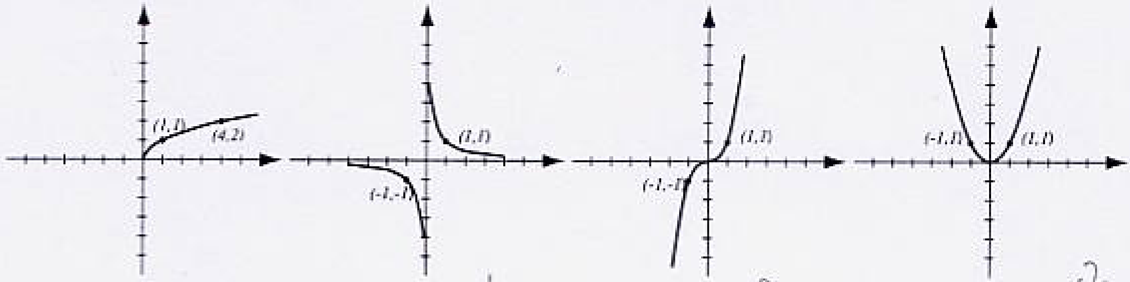
Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y=\sqrt{x}$ $\qquad$ $y=\frac{1}{x}$ $\qquad$ $y=x^3$ $\qquad$ $y=x^2$

2. (8pts) Find the equation of the line that passes through $(3,-2)$ and is perpendicular to the line $3x+2y=7$. Draw both lines in the same coordinate system.

$3x+2y=7$

$2y=7-3x$

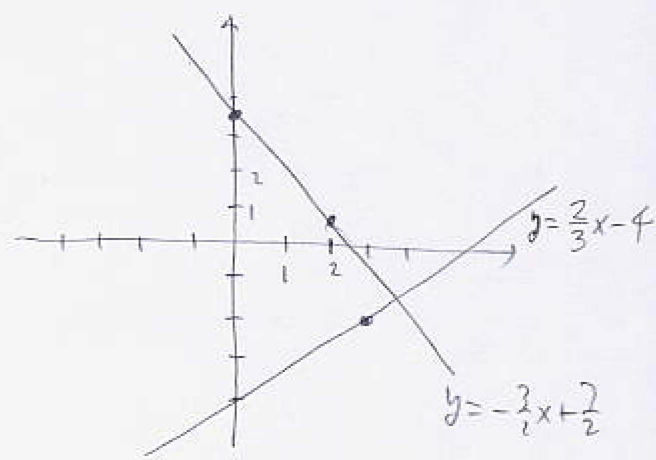
$y=-\frac{3}{2}x+\frac{7}{2}$

$m=-\frac{1}{-\frac{3}{2}}=\frac{2}{3}$

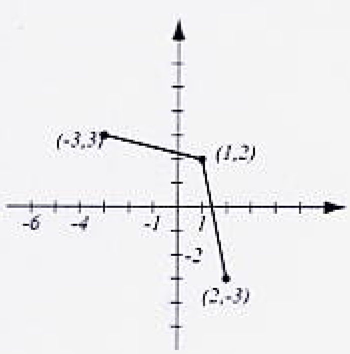
$y-(-2)=\frac{2}{3}(x-3)$

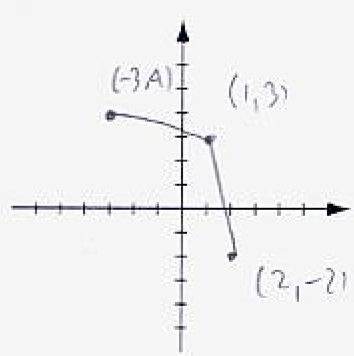
$y+2=\frac{2}{3}x-2$

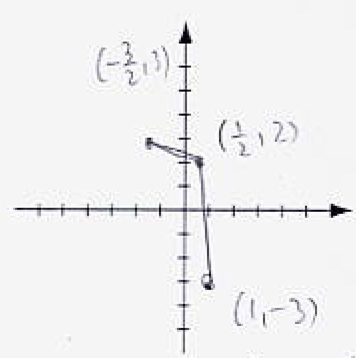
$y=\frac{2}{3}x-4$



3. (8pts) The graph of the function f is given below. On separate graphs, sketch the graphs of the functions $f(x)+1$ and $f(2x)$. Label all the relevant points.







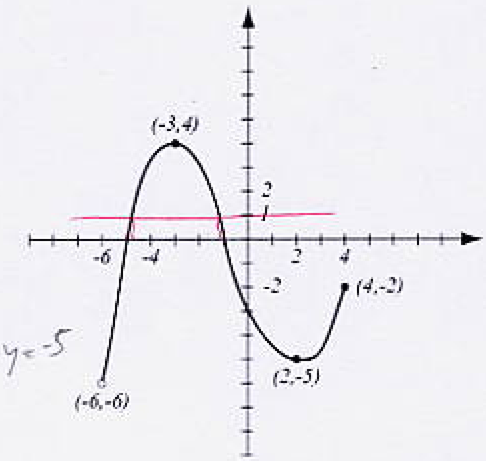
shifts up 1

$(x,y)\mapsto(x,y+1)$

stretch horizontally, factor $\frac{1}{2}$

$(x,y)\mapsto(\frac{1}{2}x,y)$

4. (10pts) Use the graph of the function f at right to answer the following questions.



a) Find $f(-3)$. $f(-3)=4$

b) What is the range of f? $(-6,4]$

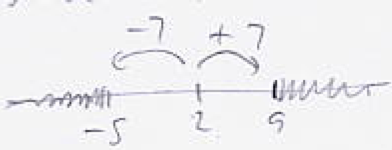
c) List the x-intercepts of the graph. $-5, -1$

d) Where does f have a local minimum? What is its value? f has a local min at $x=2$ with value $y=-5$

e) What are the solutions of the equation $f(x)=1$? $x=-1.3, -4.8$

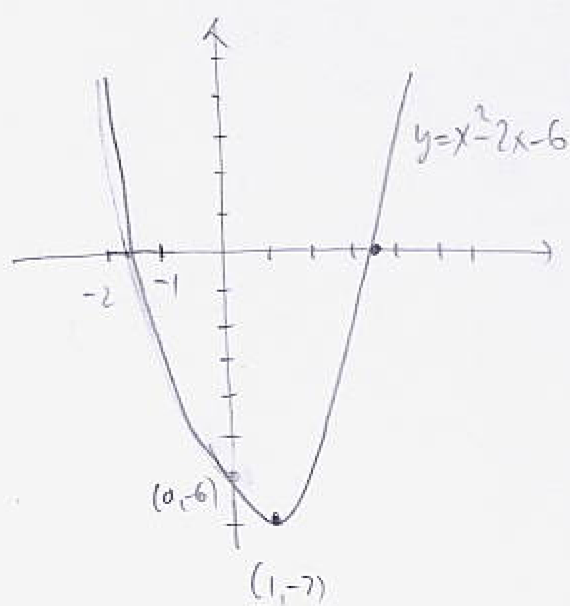
5. (6pts) Solve the inequality. Draw the solution and write it in interval form.

$|x-2| \geq 7$ distance from x to 2 ≥ 7



$(-\infty,-5] \cup [9,\infty)$

6. (12pts) The quadratic function $f(x)=x^2-2x-6$ is given. Do the following without using the calculator.
a) Find the x-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.



a) $x^2-2x-6=0$

$$x=\frac{2 \pm \sqrt{4-4\cdot 1\cdot(-6)}}{2}$$

$$=\frac{2 \pm \sqrt{28}}{2}=\frac{2 \pm 2\sqrt{7}}{2}=1 \pm \sqrt{7}$$

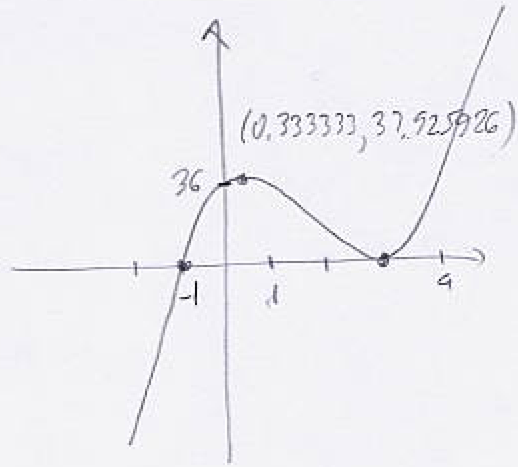
$$=3.65, -1.65$$

b) $x=-\frac{b}{2a}=-\frac{-2}{2\cdot 1}=1$ $(1,-7)$

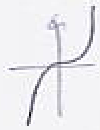
$y=f(1)=1-2-6=-7$

7. (14pts) Consider the polynomial $P(x) = 4(x-3)^2(x+1)$. Answer the following (decimal answers should have accuracy to two decimal places).
a) Find the x-intercepts of the graph and the y-intercept.
b) What is the end behavior of the polynomial?
c) Find the turning points of P.
d) Sketch the graph of the function on paper. Make sure scale is marked and all features you found in a)-c) are indicated.

a) $x=3,\ x=-1$ x-int
$P(0) = 4\cdot 9\cdot 1 = 36$



b) $P(x)$ behaves like $4x^2x = 4x^3$, will look like



c) Local min at $(3,0)$
Local max at

8. (8pts) Simplify and write the answer so all exponents are positive:

$$\frac{(2x)^4(x^{-3}y^5)^3}{(xy)^{-4}(10y)^2} = \frac{16x^4x^{-9}y^{15}}{x^{-4}y^{-4}\,100\,y^2} = \frac{4x^{-5}y^{15}}{x^{-4}y^{-2}\cdot 25} = \frac{4x^{-1}y^{17}}{25} = \frac{4y^{17}}{25x}$$

9. (8pts) Simplify.

$$\frac{x+1}{x^2+4x-5} + \frac{2x-1}{x^2+10x+25} = \frac{x+1}{(x+5)(x-1)} + \frac{2x-1}{(x+5)(x+5)} = \frac{(x+1)(x+5)+(2x-1)(x-1)}{(x+5)(x-1)(x+5)}$$

$$= \frac{x^2+6x+5+2x^2-3x+1}{(x+5)(x-1)(x+5)} = \frac{3x^2+3x+6}{(x+5)^2(x-1)} = \frac{3(x^2+x+2)}{(x+5)^2(x-1)}$$

prod = 2
sum = 1
can't factor

10. (9pts) Let $f(x) = \dfrac{2x}{5x-1}$.

a) Find $f^{-1}(x)$.

b) Find the range of f.

$y = \dfrac{2x}{5x-1}$ solve for x

$(5x-1)y = 2x$

$5xy - y = 2x \quad |-2x+y$

$5xy - 2x = y$

$x(5y-2) = y$

$x = \dfrac{y}{5y-2} \qquad f^{-1}(y) = \dfrac{y}{5y-2}$

b) range of f = domain of f^{-1}

can't have $5y-2=0$

$y = \dfrac{2}{5}$

$(-\infty, \frac{2}{5}) \cup (\frac{2}{5}, \infty)$

11. (8pts) Solve the equation. $e^{x+3} = 4^{2x-1}$

$e^{x+3} = 4^{2x-1} \quad | \ln$

$\ln e^{x+3} = \ln 4^{2x-1}$

$x+3 = (2x-1)\ln 4$

$x+3 = 2x\ln 4 - \ln 4 \quad |-2x\ln 4 - 3$

$x - 2x\ln 4 = -3 - \ln 4$

$x(1-2\ln 4) = -3 - \ln 4$

$x = \dfrac{-3-\ln 4}{1-2\ln 4} = \dfrac{3+\ln 4}{2\ln 4+1} = 1.162675$

12. (5pts) If $\log_a 9 = 1.662353$ and $\log_a 2 = 0.831176$, find (show how you obtained your numbers):

$\log_a 18 = \log_a(2 \cdot 9)$

$= \log_a 2 + \log_a 9$

$= 1.662353 + 0.831176$

$= 2.493529$

$\log_a \frac{8}{9} = \log_a 2^3 - \log_a 9 = 3\log_a 2 - \log_a 9$

$= 3 \cdot 0.831176 - 1.662353 =$

$= 0.831175$

13. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

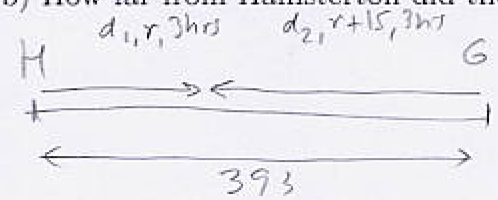
$\log_5 \dfrac{x^3}{25y^4} = \log_5 x^3 - \log_5(25y^4) = 3\log_5 x - (\log_5 25 + \log_5 y^4)$

$= 3\log_5 x - 2 - 4\log_5 y$

14. (14pts) A 393-mile-long road joins cities Hamsterton and Gerbilville. At the same time, one car leaves Hamsterton and drives toward Gerbilville, and another car, driving 15mph faster than the first car, leaves Gerbilville and drives toward Hamsterton. After 3 hours they meet on the road.

a) What are the speeds of the cars?

b) How far from Hamsterton did they meet?



a) $d_1 = r \cdot 3$

$d_2 = (r+15)\cdot 3$

$d_1 + d_2 = 393$

$r\cdot 3 + (r+15)\cdot 3 = 393$

$3r + 3r + 45 = 393 \quad | -45$

$6r = 348$

$r = \frac{348}{6} = 58$ mph

other car: 73 mph

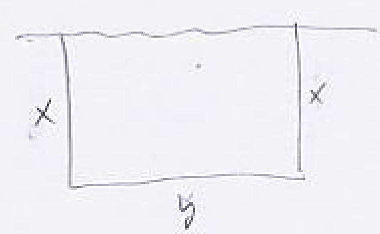
b) $3 \cdot 58 = 174$ miles from Hamsterton

15. (14pts) Sharon has 4000m of fencing and wishes to enclose a rectangular field that borders a river, where she does not fence the side along the river.

a) Express the area of the enclosure as a function of the length of one of the sides. What is the domain of this function?

b) Sketch the graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). How should the field be sized so that its area is maximal?

a)



$2x + y = 4000$

$y = 4000 - 2x$

$A = xy = x(4000 - 2x)$

$A(x) = -2x^2 + 4000x$

Domain: must have $x \geq 0$, $y \geq 0$

Domain: must have $x \geq 0$, $y \geq 0$ $\quad 4000 - 2x \geq 0$

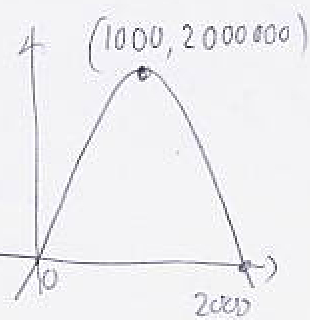
$4000 \geq 2x$

$x \leq 2000$

Domain $= [0, 2000]$

b) $x(4000 - 2x) = 0$

$x = 0$ or 2000

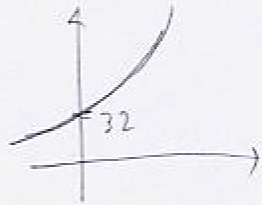


vertex is at $x = 1000$

Area is greatest for the field 1000×2000 ($4000 - 2 \cdot 1000$) has area 2,000,000.

16. (12pts) In 1994, the population of Chinchilla City was 32,000 and has since then grown according to the formula $P(t) = P_0 e^{kt}$, with a 2.5% exponential growth rate.
a) Write the function that describes the population at time t years since 1994. Graph it on paper.
b) Find the population in the year 2005.
c) When will Chinchilla City reach population 50,000?

a) $P(t) = 32\, e^{0.025t}$



b) $P(11) = 32\, e^{0.025 \cdot 11} = 42.128982$

$\uparrow$ 2005 - 1994

About 42,129 people

c) $50 = 32 e^{0.025t} \quad | \div 32$

$\frac{50}{32} = e^{0.025t} \quad | \ln$

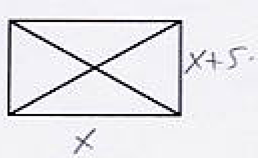
$\ln \frac{25}{16} = \ln e^{0.025t}$

$\ln \frac{25}{16} = 0.025t$

$t = \frac{\ln \frac{25}{16}}{0.025} = 17.851484$

About 18 years, so in 2012

Bonus. (15pts) 40 meters of fencing were used to enclose a rectangular area and divide it into four parts by fencing the diagonals of the rectangle. If one side of the rectangle is five meters longer than the other side, what are the dimensions of the rectangle?



$2\left(x + x+5 + \sqrt{x^2 + (x+5)^2}\right) = 40 \quad | \div 2$

$2x + 5 + \sqrt{x^2 + (x+5)^2} = 20$

$\sqrt{x^2 + x^2 + 10x + 25} = 15 - 2x \quad |^2$

$2x^2 + 10x + 25 = 225 - 60x + 4x^2$

$2x^2 - 70x + 200 = 0$

$x^2 - 35x + 100 = 0$

$x = \frac{35 \pm \sqrt{1225 - 4 \cdot 1 \cdot 100}}{2 \cdot 1}$

$= \frac{35 \pm \sqrt{825}}{2}$

$= \frac{35 \pm 5\sqrt{33}}{2}$

$= 31.861407,\ 3.138593$

too big, all fence adds up to 40

$\boxed{x = 3.138593 \text{ is the solution}}$