College Algebra — Exam 4
MAT 140, Fall 2012 — D. Ivanšić

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Show all your work!

1. (8pts) Evaluate without using the calculator:

$\log_9 81 = 2$ ($9^? = 81$)

$\log_{49} \frac{1}{7} = -\frac{1}{2}$ ($49^? = \frac{1}{7}$)

$\log_a \sqrt[3]{a^4} = \frac{4}{3}$ ($a^? = a^{\frac{4}{3}}$)

$\log_{16} 64 = \frac{3}{2}$ (think root) ($16^? = 64$, $(\sqrt{16})^3 = 4^3 = 64$)

2. (4pts) Use your calculator to find $\log_4 65$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_4 65 = \frac{\log 65}{\log 4} = 3.011184$

3. (5pts) If $\log_a 7 = 1.180299$ and $\log_a 2 = 0.420431$, find (show how you obtained your numbers):

$\log_a \frac{7}{2} = \log_a 7 - \log_a 2$
$= 1.180299 - 0.420431$
$= 0.759868$

$\log_a 56 = \log_a (8 \cdot 7) = \log_a (2^3 \cdot 7)$
$= \log_a 2^3 + \log_a 7 = 3\log_a 2 + \log_a 7$
$= 3 \cdot 0.420431 + 1.180299 = 2.491592$

4. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_3 \frac{x^4}{9\sqrt[4]{y^5}} = \log_3 x^4 - \log_3 9 - \log_3 y^{\frac{5}{4}}$
$= 4\log_3 x - 2 - \frac{5}{4}\log_3 y$

5. (13pts) Write as a single logarithm. Simplify if possible.

$3\log(x^4y^3) - 2\log(x^3y^4) = \log(x^4y^3)^3 - \log(x^3y^4)^2 =$
$= \log \frac{x^{12}y^9}{x^6y^8} = \log(x^6y)$

$2\log_7(x^2+2x-15) + \log_7(x+5) - 3\log_7(x-3) = \log_7 (x^2+2x-15)^2 + \log_7(x+5) - \log_7(x-3)^3$ ($x^2+2x-15 = (x+5)(x-3)$)
$= \log_7 \frac{((x+5)(x-3))^2 (x+5)}{(x-3)^3} = \log_7 \frac{(x+5)^2(x-3)^2(x+5)}{(x-3)^3} = \log_7 \frac{(x+5)^3}{x-3}$

6. (8pts) How much should you invest in an account bearing 3.32%, compounded monthly, if you wish to have \$9,000 in three years?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

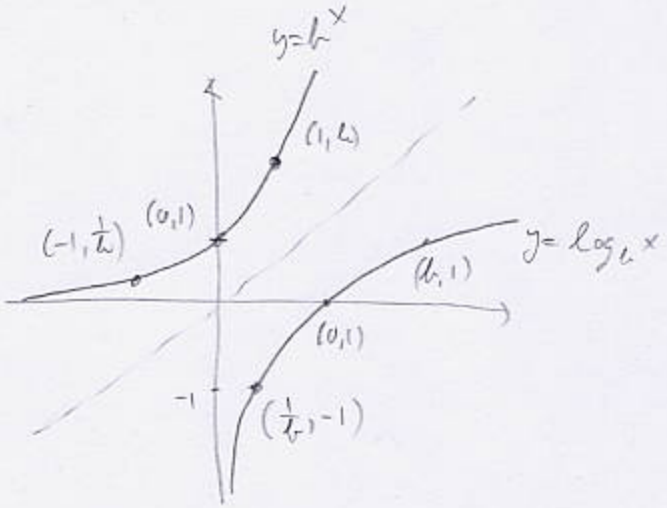
$9000 = P\left(1+\frac{0.0332}{12}\right)^{12\cdot 3}$

$9000 = P\cdot(1.00276..)^{36}$

$9000 = P\cdot 1.1045.. \quad 1 \div 1.1045$

$P = \frac{9000}{1.1045..} = 8147.92$

7. (9pts) Let $f(x) = b^x$, $b > 1$.
a) Draw the graph of the exponential function. Indicate three points on this graph.
b) Use the graph of f to draw the graph of its inverse $f^{-1}(x) = \log_b x$. Indicate three points on this graph.
c) What is the range of $f^{-1}(x)$?



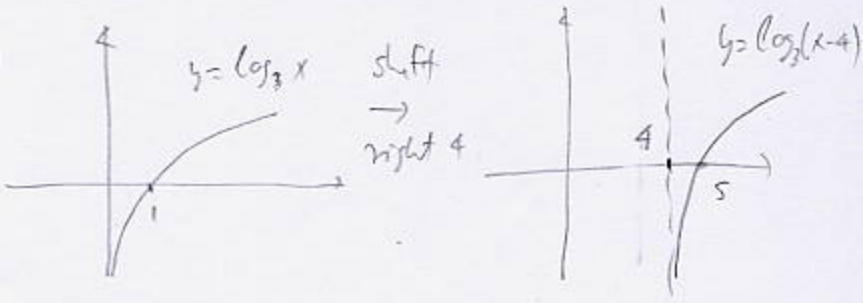
8. (8pts) Let $f(x) = 2\log_3(x-4)$.
a) What is the domain of f?
b) Explain how you transform the graph of $\log_3 x$ in order to get the graph of f. Indicate the x-intercept and any asymptotes.

a) Must have: $x-4>0$

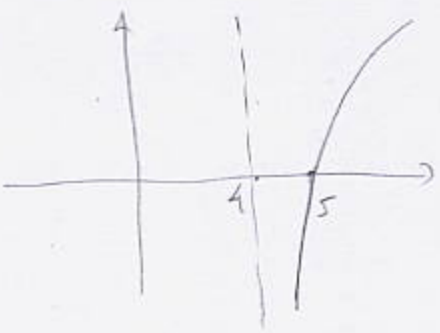
$x>4$

Domain: $(4,\infty)$

b)



stretch vertically → factor 2



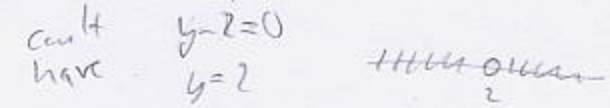
9. (9pts) Let $f(x) = \dfrac{2x}{x-3}$, $x \geq 0$.

a) Find the formula for f^{-1}.

b) Find the range of f.

b) Range of f = domain of f^{-1}

can't have $y-2=0$, $y=2$



Range: $(-\infty, 2) \cup (2, \infty)$

a) $y = \dfrac{2x}{x-3}$

$y(x-3) = 2x$

$yx - 3y = 2x \quad | -2x + 3y$

$yx - 2x = 3y$

$x(y-2) = 3y$

$x = \dfrac{3y}{y-2} \qquad f^{-1}(y) = \dfrac{3y}{y-2}$

Solve the equations.

10. (8pts) $\left(\dfrac{1}{6}\right)^{3x-1} = 36^{3-5x}$

$(6^{-1})^{3x-1} = (6^2)^{3-5x}$

$6^{-3x+1} = 6^{6-10x}$

$-3x + 1 = 6 - 10x$

$7x = 5$

$x = \dfrac{5}{7}$

11. (10pts) $\log_8(x-2) + \log_8(x+6) = 2\log_8(x+4)$

$\log_8((x-2)(x+6)) = \log_8(x+4)^2$

$(x-2)(x+6) = (x+4)^2$

$x^2 + 4x - 12 = x^2 + 8x + 16$

$4x - 12 = 8x + 16 \quad | -4x - 16$

$-28 = 4x$

$x = -7$

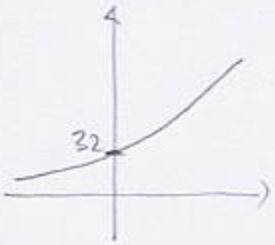
Check:

$\log_8(-9) + \log_8(-1) = 2\log_8(3)$

not defined so no solution

12. (12pts) In 1994, the population of Chinchilla City was 32,000 and has since then grown according to the formula $P(t) = P_0 e^{kt}$, with a 2.5% exponential growth rate.
a) Write the function that describes the population at time t years since 1994. Graph it on paper.
b) Find the population in the year 2005.
c) When will Chinchilla City reach population 50,000?

a) $P(t) = 32\,e^{0.025t}$



b) $P(11) = 32 \cdot e^{0.025 \cdot (11)} = 42.128982$,
t
2005 − 1994
about 42,129 people

c) $50 = 32\,e^{0.025t} \mid \div 32$
$\frac{50}{32} = e^{0.025t} \mid \ln$
$\ln \frac{25}{16} = \ln e^{0.025t}$
$\ln \frac{25}{16} = 0.025t$
$t = \frac{\ln \frac{25}{16}}{0.025} = 17.851489$
About 18 years, so sometime in 2012.

Bonus (10pts) If you invest in an account bearing 7% interest, compounded weekly, how long will it take until your money doubles? *(Hint: the deposit amount doesn't matter.)*

$A = P\left(1 + \frac{r}{n}\right)^{nt}$
Take $P = 1$, $A = 2$
$2 = \left(1 + \frac{0.07}{52}\right)^{52t}$
$2 = (1.001346\ldots)^{52t} \mid \ln$
$\ln 2 = \ln(1.001346\ldots)^{52t}$
$\ln 2 = 52t \ln(1.001346\ldots)$
$t = \frac{\ln 2}{52 \ln(1.001346\ldots)} = 9.908766$
Almost 10 years