College Algebra — Exam 3
MAT 140, Fall 2012 — D. Ivanšić

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Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (4pts) $(3-2i)(5-i) =$ $15 - 3i - 10i + 2i^2 = 15 - 13i - 2 = 13 - 13i$

2. (6pts) $\dfrac{1-i}{-2+i} = \dfrac{1-i}{-2+i} \cdot \dfrac{-2-i}{-2-i} = \dfrac{-2-i+2i+i^2}{(-2)^2 - i^2} = \dfrac{-2+i-1}{4-(-1)} = \dfrac{-3+i}{5}$

3. (4pts) Simplify and justify your answer.

$i^{1115} =$ $i^{1112} \cdot i^3 = (\underbrace{i^4}_{=1})^{278} \cdot i^3 = i^3 = -i$

$1115 \div 4 = 278$, remainder 3 $\quad 278 \cdot 4 = 1112$

Solve the equations:

4. (6pts) By completing the square:

$x^2 + 8x = -2$ $\quad 1 + 4^2$

$x^2 + 2 \cdot 4 \cdot x + 4^2 = -2 + 4^2$

$(x+4)^2 = 14$ $\quad 1\sqrt{\ }$

$x + 4 = \pm\sqrt{14}$

$x = -4 \pm \sqrt{14}$

5. (4pts) $|3x - 1| = 4$

$3x - 1 = 4$ or $3x - 1 = -4$

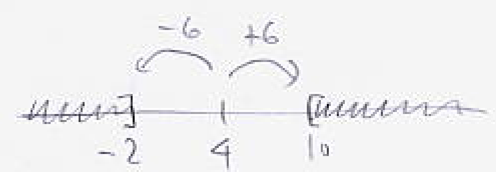
$3x = 5 \qquad 3x = -3$

$x = \dfrac{5}{3}$ or $x = -1$

6. (6pts) Solve the inequality. Draw the solution and write it in interval form.

$|x - 4| \geq 6$

distance from x to $4 \geq 6$



$(-\infty, -2] \cup [10, \infty)$

7. (14pts) The quadratic function $f(x) = x^2 - 4x - 5$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

d) What is the range of the function?

a) x-int:

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

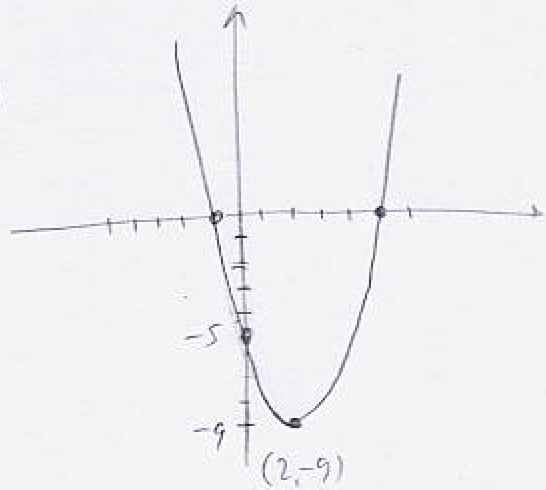
$x = -1, 5$

y-int: $f(0) = -5$

b) $x = -\frac{b}{2a} = -\frac{-4}{2 \cdot 1} = 2$

$f(2) = 4 - 8 - 5 = -9$

c)



d) From picture: range $= [-9, \infty)$

Solve the equations:

8. (8pts) $3x^4 + 2x^2 - 8 = 0$

Let $u = x^2$

$3u^2 + 2u - 8 = 0$

$u = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 3 \cdot (-8)}}{2 \cdot 3}$

$= \frac{-2 \pm \sqrt{4+96}}{6} = \frac{-2 \pm 10}{6}$

$= -\frac{12}{6}, \frac{8}{6} = -2, \frac{4}{3}$

$x^2 = -2 \qquad x^2 = \frac{4}{3}$

$x = \pm\sqrt{2}\, i \qquad x = \pm\sqrt{\frac{4}{3}}$

9. (8pts) $\sqrt{3x+4} = \sqrt{2x-5} - 2 \quad |^2$

$3x + 4 = 2x - 5 - 2 \cdot 2\sqrt{2x-5} + 4$

$3x + 4 = 2x - 1 - 4\sqrt{2x-5} \quad -2x+1$

$x + 5 = 4\sqrt{2x-5} \quad |^2$

$x^2 + 10x + 25 = 16(2x-5)$

$x^2 + 10x + 25 = 32x - 80 \quad |-32x+80$

$x^2 - 22x + 105 = 0$

$(x-7)(x-15) = 0 \qquad x = 7, 15$

Check: $\sqrt{21+4} \overset{?}{=} \sqrt{14-5} - 2$

$\sqrt{25} \overset{?}{=} \sqrt{9} - 2$ no

$\sqrt{45+4} \overset{?}{=} \sqrt{30-5} - 2$

$\sqrt{49} \overset{?}{=} \sqrt{25} - 2$ no

No solution.

10. (14pts) The polynomial $f(x) = (x-3)^2(x+4)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with what you learned in a) and b) to sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

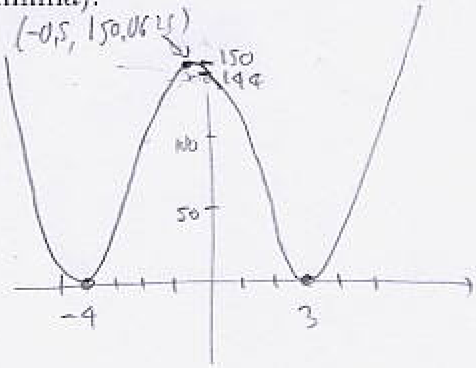
a) $x^2 \cdot x^2 = x^4$, like x^4 ∪

b)

zeroes	3	−4
multiplicities	2	2

y-int: $f(0) = (-3)^2 \cdot 4^2 = 144$

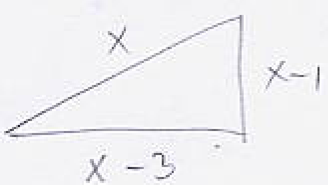
c)



d) Local minima: $x=-4,\ y=0$
$x=3,\ y=0$

Local max: $x=-0.5,\ y=150.0625$

11. (12pts) In a right triangle, one side of the triangle is 3cm shorter than the hypothenuse and the other side is 1cm shorter than the hypothenuse. What is the length of the hypothenuse?



$(x-1)^2 + (x-3)^2 = x^2$

$x^2 - 2x + 1 + x^2 - 6x + 9 = x^2$

$2x^2 - 8x + 10 = x^2$

$x^2 - 8x + 10 = 0$

$x = \dfrac{8 \pm \sqrt{(-8)^2 - 4\cdot 1\cdot 10}}{2\cdot 1} = \dfrac{8 \pm \sqrt{64-40}}{2} = \dfrac{8 \pm \sqrt{24}}{2} = \dfrac{8 \pm 2\sqrt{6}}{2}$

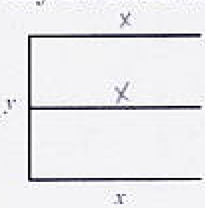
$= 4 \pm \sqrt{6} = 6.449490,\ \underbrace{1.550510}$

doesn't fit context, x−3 would be a negative number.

12. (14pts) Truck mechanic Rodrigo wishes to build two side-by-side repair bays for trucks separated by a wall (see picture). One side of the repair bays does not need a wall (doors come there). Rodrigo has enough money to build 600 feet of walls, and he wants to build a bay with maximal area.

a) Express the area of the enclosure as a function of the length of one of the sides. What is the domain of this function?

b) Sketch the graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). How should the repair bay be sized so that its area is maximal?



a) $3x+y=600$

$y=600-3x$

$A=xy=x(600-3x)=-3x^2+600x$

(a quadratic function)

Domain: must have $x>0$, $y>0$

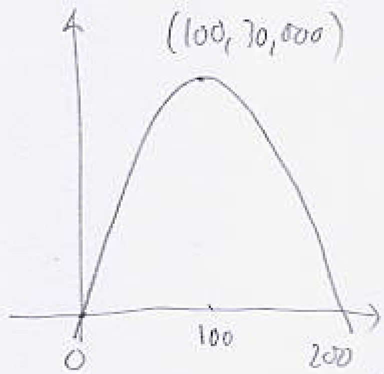
$600-3x>0$

$600>3x$

$x<200$

Domain: $[0,200]$

b)



vertex: $x=-\frac{600}{2(-3)}=100$

$A(100)=100\cdot 300=30{,}000$

Dimensions that attain max area:

100×300

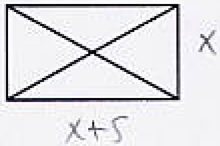
$x \quad y$

x-int: $x=0$

$600-3x=0$, $x=200$

$A(x)=x(600-3x)$

Bonus. (10pts) 40 meters of fencing were used to enclose a rectangular area and divide it into four parts by fencing the diagonals of the rectangle. If one side of the rectangle is five meters longer than the other side, what are the dimensions of the rectangle?



$2x+2(x+5)+2\sqrt{x^2+(x+5)^2}=40 \quad | \div 2$

$x+x+5+\sqrt{x^2+x^2+10x+25}=20 \quad | -2x-5$

$\sqrt{2x^2+10x+25}=15-2x \quad |^2$

$2x^2+10x+25=225-60x+4x^2 \quad | -2x^2-10x-25$

$2x^2-70x+200=0$

$x^2-35x+100=0$

$x=\frac{35\pm\sqrt{35^2-4\cdot 1\cdot 100}}{2}=\frac{35\pm\sqrt{1225-400}}{2}=\frac{35\pm\sqrt{825}}{2}=\frac{35\pm 5\sqrt{33}}{2}$

Sol: 31.861407 ← too big, all fence adds up to 40

$\boxed{3.138593 \text{ solution}}$