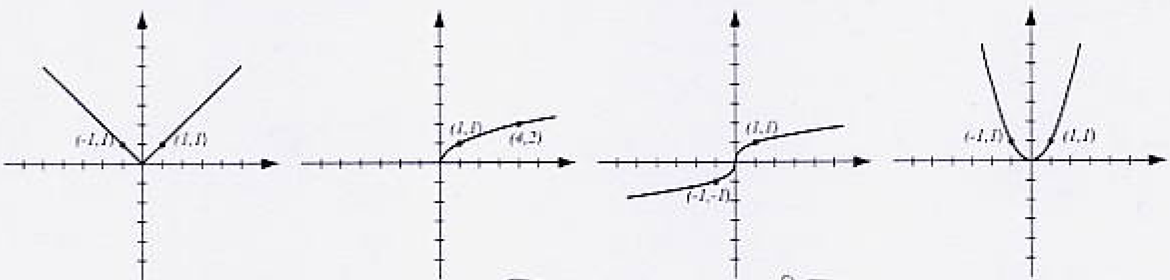
College Algebra — Exam 2 **MAT 140, Fall 2012 — D. Ivanšić**	**Name:** Saul Ocean *Show all your work!*

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y = |x|$ $\qquad$ $y = \sqrt{x}$ $\qquad$ $y = \sqrt[3]{x}$ $\qquad$ $y = x^2$

2. (10pts) Find the equation of the line L (in form $y = mx + b$) that passes through points $(-3, 2)$ and $(2, 7)$. Then find the equation of line that passes through $(1, 1)$ and is perpendicular to L.

$m = \frac{7-2}{2-(-3)} = \frac{5}{5} = 1$

$y - 7 = 1 \cdot (x-2)$

$y = x - 2 + 7$

$y = x + 5$ line L

slope of reciprocal line is $-\frac{1}{1} = -1$

$y - 1 = (-1)(x-1)$

$y - 1 = -x + 1$

$y = -x + 2$ perp. to L

Solve the inequalities. Write your solution in interval notation.

3. (6pts) $8 \le 5x - 2 < 16$ $\quad |+2$

$10 \le 5x < 18 \quad |\div 5$

$2 \le x < \frac{18}{5}$

$[2, \frac{18}{5})$

4. (7pts) $3x + 4 < 5$ or $3x - 2 \ge 7$

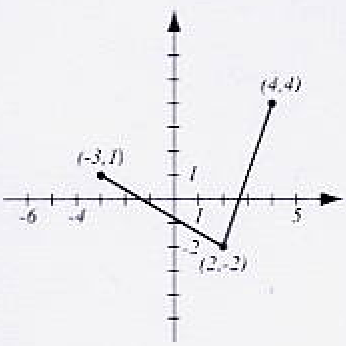
$3x < 1 \qquad 3x \ge 9$

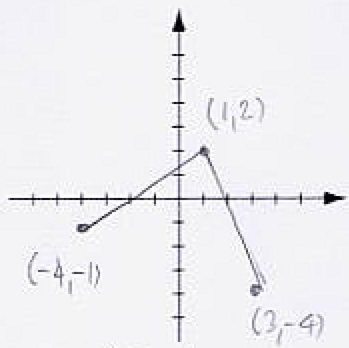
$x < \frac{1}{3}$ or $x \ge 3$

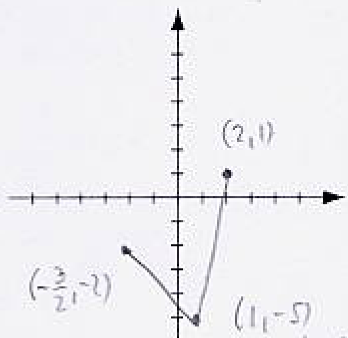
$\frac{1}{3} \qquad 3$

$(-\infty, \frac{1}{3}) \cup [3, \infty)$

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x+1)$ and $f(2x)-3$ and label all the relevant points.







shift left 1, reflect in x-axis

stretch horizontally, factor $= \frac{1}{2}$, shift down 3

6. (20pts) Let $f(x) = \dfrac{2}{\sqrt{x+6}}$, $g(x) = \sqrt{4-3x}$.

Find the following (simplify where possible):

$(f-g)(1) = f(1) - g(1)$

$= \dfrac{2}{\sqrt{7}} - \sqrt{1} = \dfrac{2}{\sqrt{7}} - 1$

$\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{\frac{2}{\sqrt{x+6}}}{\sqrt{4-3x}} = \dfrac{2}{\sqrt{x+6}} \cdot \dfrac{1}{\sqrt{4-3x}}$

$= \dfrac{2}{\sqrt{(x+6)(4-3x)}}$

$(g \circ f)(0) = g(f(0)) = g\left(\dfrac{2}{\sqrt{6}}\right) = \sqrt{4 - 3 \cdot \frac{2}{\sqrt{6}}}$

$= \sqrt{4 - \frac{6}{\sqrt{6}}} = \sqrt{4 - \sqrt{6}}$

$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{4-3x}\right)$

$= \dfrac{2}{\sqrt{\sqrt{4-3x} + 6}}$

The domain of f

Must have $x+6 > 0$

$x > -6$

$(-6, \infty)$

(can't have $x+6=0$)

The domain of g

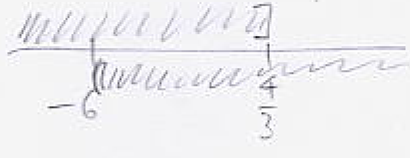
Must have $4-3x \geq 0$

$4 \geq 3x$

$x \leq \frac{4}{3}$

$(-\infty, \frac{4}{3}]$

The domain of $(f+g)(x)$

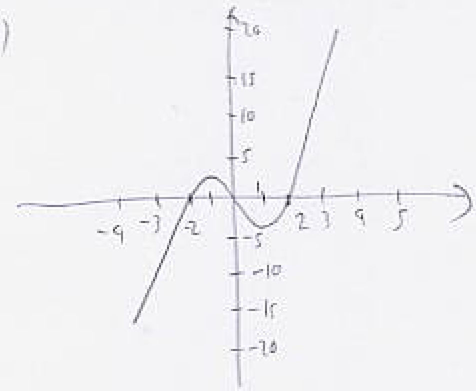


Overlap = domain of $f+g$

$= (-6, \frac{4}{3}]$

7. (17pts) Let $f(x) = x^3 - 4x$ (answer with 6 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.
b) Determine algebraically whether f is even, odd, or neither. Verify your answer further by examining the graph.
c) Find the local maxima and minima for this function.
d) State the intervals where the function is increasing and where it is decreasing.

a)



c) $f(1.154695) = -3.079201$ is a local min

$f(-1.154695) = 3.079201$ is a local max

d) Increasing on

$(-\infty, -1.154695) \cup (1.154695, \infty)$

Decreasing on

$(-1.154695, 1.154695)$

b) $f(-x) = (-x)^3 - 4(-x)$

$= -x^3 + 4x = -f(x)$

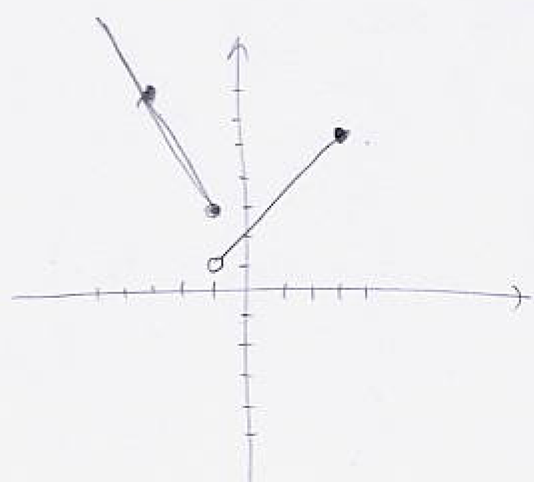
f is odd - which can also be seen from graph, - it is symm about origin.

8. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} -2x+1, & \text{if } x \le -1 \\ x+2, & \text{if } -1 < x \le 5. \end{cases}$$

x	$-2x+1$
-1	3
-3	7

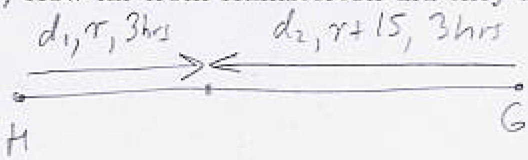
x	$x+2$
-1	1
3	5



9. (14pts) A 393-mile-long road joins cities Hamsterton and Gerbilville. At the same time, one car leaves Hamsterton and drives toward Gerbilville, and another car, driving 15mph faster than the first car, leaves Gerbilville and drives toward Hamsterton. After 3 hours they meet on the road.

a) What are the speeds of the cars?

b) How far from Hamsterton did they meet?



Let r = speed of car leaving Hamsterton

$d_1 = r \cdot 3$

$d_2 = (r+15) \cdot 3$

$d_1 + d_2 = 393$

$3r + 3(r+15) = 393$

$6r + 45 = 393 \quad | -45$

$6r = 348 \quad | \div 6$

$r = 58$

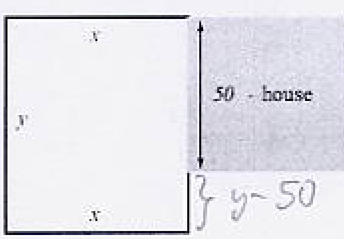
a) Cars travel at speeds 58mph, 73mph

b) $58 \cdot 3 = 174$ miles from Hamsterton

Bonus. (10pts) Manuel has 400ft of fencing that he will use to enclose a rectangular pen for his rabbit. The pen will use the entire 50ft wall of his house as part of a side of the pen (see the picture). Follow the steps below to find the dimensions of the pen that has the greatest area.

a) Write the area $A(x)$ as a function of x. What is the domain of A?

b) Graph $A(x)$ in order to find the maximum. What are the dimensions of the enclosure that has the greatest area?



a) Fencing used $= x + x + y + y - 50 = 2x + 2y - 50$

$2x + 2y - 50 = 400$

$2x + 2y = 450 \quad | \div 2$

$x + y = 225$

$y = 225 - x$

Must have $y \geq 50$

$225 - x \geq 50 \quad | +x-50$

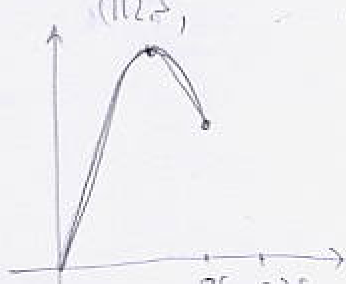
$175 \geq x \geq 0$

Domain = $[0, 175]$

$A(x) = xy = x(225-x) = -x^2 + 225x$

112 · 113
112
336
12656

b)



Greatest area is for 112.5×112.5.

Greatest area is 12656.25