College Algebra — Exam 1
MAT 140, Fall 2012 — D. Ivanšić

Name: Saul Ocean

Show all your work!

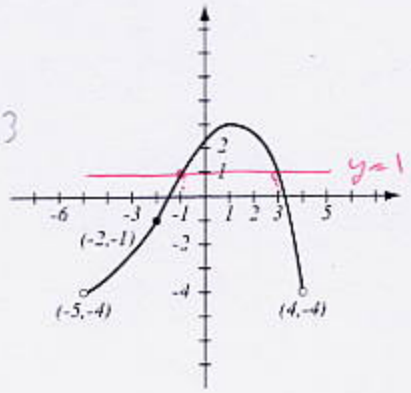
1. (8pts) Use the graph of the function f at right to answer the following questions.

a) Find $f(-2)$ and $f(0)$. $f(-2)=-1$, $f(0)=2.3$

b) What is the domain of f? $(-5, 4)$

c) What is the range of f? $(-4, 3]$

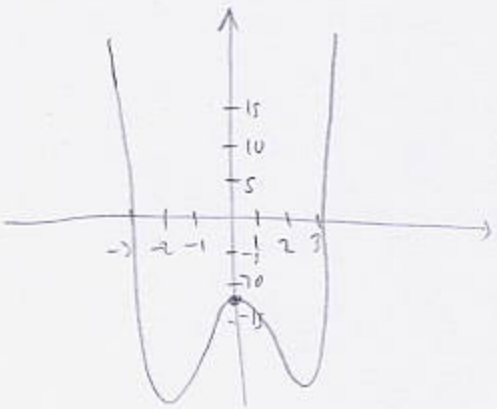
d) What are the solutions of the equation $f(x) = 1$? $x=-1, 3$



2. (10pts) Use your calculator to accurately sketch the graph of $y = x^4 - 8x^2 - 11$. Draw the graph here, and indicate units on the axes. Find all the $x-$ and $y-$intercepts (accuracy: 6 decimal points).

x-int: $-3.032516, 3.032516$

y-int: -11



3. (4pts) Convert to scientific notation or a decimal number:

$27,110 =$ 2.711×10^4 $3.159 \times 10^{-5} =$ 0.00003159

Use formulas to expand:

4. (4pts) $(3x-2)^2 =$ $(3x)^2 - 2 \cdot 3x \cdot 2 + 2^2 = 9x^2 - 12x + 4$

5. (4pts) $(x^2-y)(x^2+y) =$ $(x^2)^2 - y^2 = x^4 - y^2$

6. (6pts) $(x+5)^3 =$ $x^3 + 3 \cdot x^2 \cdot 5 + 3 \cdot x \cdot 5^2 + 5^3 = x^3 + 15x^2 + 75x + 125$

Simplify, showing intermediate steps.

7. (2pts) $\sqrt{63} = \sqrt{9 \cdot 7} = 3\sqrt{7}$

8. (5pts) $\sqrt[3]{40x^4} = \sqrt[3]{5 \cdot 8 \cdot x^3 \cdot x}$

$= 2x\sqrt[3]{5x}$

9. (7pts) $\dfrac{\sqrt[4]{324x^5y^{11}}}{\sqrt[4]{2xy^2}} = \sqrt[4]{\dfrac{324x^5y^{11}}{2xy^2}} = \sqrt[4]{162x^4y^9} = \sqrt[4]{2 \cdot 81 \cdot x^4 \cdot (y^2)^4 \cdot y}$

$= \sqrt[4]{2} \cdot 3 \cdot |x| \cdot |y^2| \sqrt[4]{y} = 3|x|y^2\sqrt[4]{2y}$

≥ 0

10. (8pts) Simplify.

$$\frac{2x-1}{x^2-49} - \frac{4}{x^2+4x-21} = \frac{2x-1}{(x-7)(x+7)} - \frac{4}{(x+7)(x-3)} = \frac{(2x-1)(x-3) - 4(x-7)}{(x-7)(x+7)(x-3)}$$

$$= \frac{2x^2-7x+3-4x+28}{(x-7)(x+7)(x-3)} = \frac{2x^2-11x+31}{(x-7)(x+7)(x-3)}$$

doesn't factor

prod = 62	$\pm(1,62)$	$\pm(2,31)$
sum = −11	no	no

11. (8pts) Simplify. Express answers first in terms of positive exponents, then convert to root notation.

$$\frac{(64x^{-2}y^6)^{\frac{2}{3}}}{\left(2x^{-\frac{3}{5}}y^{\frac{3}{4}}\right)^4} = \frac{64^{\frac{2}{3}}\, x^{-2\cdot\frac{2}{3}}\, y^{6\cdot\frac{2}{3}}}{2^4\, x^{-\frac{3}{5}\cdot 4}\, y^{\frac{3}{4}\cdot 4}} = \frac{\left(\sqrt[3]{64}\right)^2 x^{-\frac{4}{3}} y^4}{16\, x^{-\frac{12}{5}} y^3} = \frac{16\, x^{-\frac{4}{3}+\frac{12}{5}}\, y^{4-3}}{16}$$

$$= x^{\frac{-20+36}{15}} y^{1} = x^{\frac{16}{15}} y = \frac{\sqrt[15]{x^{16}}}{y^{16}} = \frac{\sqrt[15]{x^{16}}}{y^{16}}$$

12. (6pts) Rationalize the denominator. 8

$\frac{4\sqrt{2}-5}{3-\sqrt{2}}$ $\cdot \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{12\sqrt{2}-15+4\cdot\sqrt{2}^2-5\sqrt{2}}{3^2-\sqrt{2}^2} = \frac{7\sqrt{2}-7}{7} = \frac{7(\sqrt{2}-1)}{7} = \sqrt{2}-1$

13. (4pts) Solve the equation for y.

$2x+3y=c \quad |-2x$

$3y = c-2x$

$y = \frac{c-2x}{3}$

14. (8pts) Solve the equation.

$3x^2+5x = 6-x^2 \quad |+x^2-6$

$4x^2+5x-6=0$

prod $= -24$ $8,-3$

sum $= 5$

$4x^2+8x-3x-6=0$

$4x(x+2)-3(x+2)=0$

$(4x-3)(x+2)=0$

$4x-3=0 \quad x+2=0$

$x=\frac{3}{4}$ or $x=-2$

15. (4pts) Find the domain of the function $f(x) = \frac{|x-7|}{x+4}$.

Can't have $x+4=0$

$x \neq -4$

Domain $= \{x \mid x \neq -4\}$

16. (12pts) The circle whose diameter has endpoints $(5, 1)$ and $(-1, -1)$ is given.
a) Find the equation of the circle.
b) Draw the circle in the coordinate plane.
c) Is this circle the graph of a function? Why or why not?

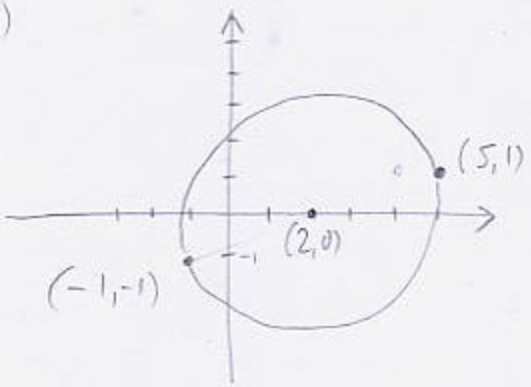
a) Center = midpoint of the two pts

$M = \left(\frac{5-1}{2}, \frac{1-1}{2}\right) = (2, 0)$

radius = distance from $(2,0)$ to $(5,1) = \sqrt{(5-2)^2 + (1-0)^2} = \sqrt{9+1} = \sqrt{10}$

$(x-2)^2 + (y-0)^2 = 10$ $\quad \sqrt{10} \approx 3.2$

b)



c) The graph fails the vertical line test, so it is not the graph of a function

Bonus (10pts) Simplify.

$$\frac{3 + \dfrac{18x^2 - 4x}{x^3 - 8}}{1 + \dfrac{10x + 44}{x^2 + 2x - 8}}$$

$x^3 - 2^3$

$$= \frac{\dfrac{3(x^3-8) + 18x^2 - 4x}{x^3 - 8}}{\dfrac{x^2 + 2x - 8 + 10x + 44}{x^2 + 2x - 8}} = \frac{\dfrac{3x^3 - 24 + 18x^2 - 4x}{(x-2)(x^2+2x+4)}}{\dfrac{x^2 + 12x + 36}{(x+4)(x-2)}}$$

$$= \frac{3x^2(x+6) - 4(x+6)}{3x^3 + 18x^2 - 4x - 24} \cdot \frac{(x+4)(x-2)}{x^2 + 12x + 36} = \frac{(3x^2-4)(x+6)(x+4)}{(x^2+2x+4)(x+6)(x+6)}$$

$$= \frac{(3x^2-4)(x+4)}{(x^2+2x+4)(x+6)}$$