

1. (30pts) A car-rental company is considering upgrading part of its fleet with vehicles from one manufacturer. Competitive proposals were submitted by Chevrolet, Dodge, Hyundai and Toyota, and the company asks its regional managers to rank the four proposals. It will decide on the manufacturer based on these rankings.

Votes	6	4	3	3	1	7	5	4	= total = 33
1st	C	C	D	D	T	T	H	H	
2nd	D	D	C	C	D	H	D	T	
3rd	H	T	H	T	H	C	T	C	
4th	T	H	T	H	C	D	C	D	

- Which choice wins the vote in a plurality election?
- Which choice wins the vote in a plurality election with elimination?
- Which choice is the pairwise comparison winner?
- Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

a) C $6+4 = 10$ wins
 D $3+3 = 6$
 T $1+7 = 8$
 H $5+4 = 9$

b) D is eliminated first

C $10+3+3 = 16$

T $8 = 8$

H $9 = 9$

T is eliminated second

C $16 + = 16$

H $9+9+9 = 17$ wins

c) Pairwise pairs:

C $10+7+4 = 21$ wins

D $6+1+5 = 12$

C $10+3+3 = 16$

H $9+7+1 = 17$ wins

C $10+3+3 = 16$

T $8+5+4 = 17$ wins

D $6+6+4+1 = 17$ wins

H $9+7 = 16$

D $6+6+4+5 = 21$ wins

T $8+4 = 12$

H $9+6+3 = 18$ wins

T $8+4+3 = 15$

It's a tie for pairwise comparison between D and H.

d) Borda points:

C $4 \cdot 10 + 3 \cdot 6 + 2 \cdot 11 + 1 \cdot 6 = 86$ wins

D $4 \cdot 6 + 3 \cdot 6 + 2 \cdot 0 + 1 \cdot 11 = 83$

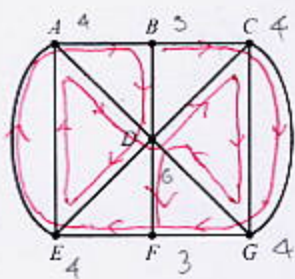
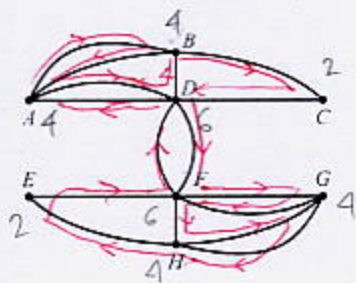
T $4 \cdot 8 + 3 \cdot 4 + 2 \cdot 12 + 1 \cdot 9 = 77$

H $4 \cdot 9 + 3 \cdot 7 + 2 \cdot 10 + 1 \cdot 7 = 84$

330

330 which equals 33×10
 $\uparrow \quad \uparrow$
 votes points per voter

2. (12pts) Determine whether each of the following graphs has an Euler path or an Euler circuit. If it does, find it, if not, explain why not. Write the sequence of vertices Euler paths or circuits go through.



Has exactly two vertices
- has an Euler path, but no circuit
BCGF EABD EADCGDF

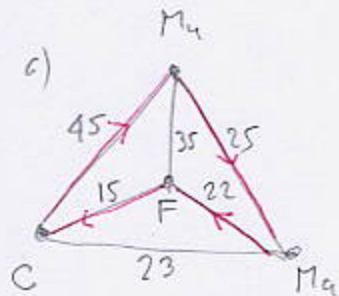
All vertices even
⇒ has an Euler circuit
ABADBCDFGFHGHEDA

3. (18pts) A soda distributor wishes to restock vending machines in Clinton, Fulton, Mayfield and Murray while minimizing total distance traveled. The table below has the distances between the cities.

- Draw a weighted graph that corresponds to this problem.
- Use the brute force method to find the route that minimizes the distance traveled. First list all the possible orders of visits with Murray the starting city.
- Use the nearest neighbor algorithm to find an approximate solution to the problem. Is it the same as in b)?

	C	F	Ma
F	15		
Ma	23	22	
Mu	45	35	25

b) $M_u C F M_a M_u: 45 + 15 + 22 + 25 = 107$
 $M_u C M_a F M_u: 45 + 23 + 22 + 35 = 125$
 $M_u F C M_a M_u: 35 + 15 + 23 + 25 = 98 \leftarrow \text{shortest}$
 $M_u F M_a C M_u: 35 + 22 + 23 + 45 = 125$
 $M_u M_a C F M_u: 25 + 23 + 15 + 35 = 98$
 $M_u M_a F C M_u: 25 + 22 + 15 + 45 = 107$



c) Pictured, length is $25 + 22 + 15 + 45 = 107$,
not the shortest route.