Mathematical Concepts — Joysheet 9
MAT 117, Fall 2012 — D. Ivanšić

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Show all your work!

Final answers should have accuracy to 6 decimal places (or 4 decimal places for table-derived answers). Show some work how the mean and standard deviation are computed. *Giving only the answer will bring you few points.*

1. (18pts) A number of bands were analyzed to determine how many hits they had over a five-year period (according to some definition of what a "hit" is). The number of hits is recorded below.
a) Find the range of the number of hits.
b) Find the mean number of hits.
c) Find the standard deviation of the number of hits.

Number of hits	Frequency (bands)
1	5
2	4
3	11
4	17
5	9
6	4
7	2
	52

a) $7-1=6$

b) $\bar{x}=\dfrac{5\cdot1+4\cdot2+11\cdot3+17\cdot4+9\cdot5+4\cdot6+2\cdot7}{5+4+11+17+9+4+2}=$

$=\dfrac{197}{52}=3.788462$

c) $5(1-3.78\_)^2+4(2-3.78\_)^2+\ldots+2(7-3.78\_)^2=112.673077$

$s=\sqrt{\dfrac{112.67\_}{51}}=\sqrt{2.20\_}=1.486363$

2. (10pts) The life-span of a certain light bulb is normally distributed with mean 2,500 hours and standard deviation 200. Use the 68-95-99.7 rule (draw a picture) to find the percentage of light-bulbs that lasted:

a) between 2,300 and 2,700 hours

68%

b) under 2,100 hours

$\frac{0.95}{2}=0.475$ $\quad 0.5-0.475=0.025,\ 2.5\%$

c) over 2,700 hours

$\frac{0.68}{2}=0.34$ $\quad 0.5-0.34=0.16,\ 16\%$

d) between 1,900 and 2,300 hours

$\dfrac{0.997}{2}-\dfrac{0.68}{2}=0.4985-0.34$

$=0.1585,\ 15.85\%$

3. (6pts) A set of data items is normally distributed with mean 23 and standard deviation 5.1. Find the data items that correspond to the z-scores given below.

a) $z = 0$

$23 + 0 \cdot 5.1$
$= 23$

b) $z = 1.3$

$23 + 1.3 \cdot 5.1$
$= 29.63$

c) $z = -2.2$

$23 - 2.2 \cdot 5.1$
$= 11.78$

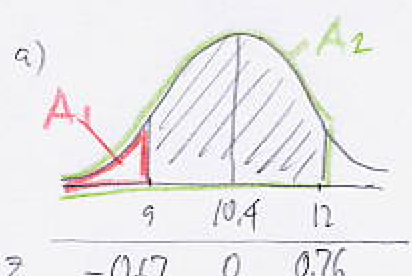
4. (4pts) Kate scored 14 points on an exam with mean 20 and standard deviation 4, and Kacie scored 43 points on a similar exam with mean 50 and standard deviation 5. Use z-scores to determine who did worse.

$$z_{Kate} = \frac{14-20}{4} = -1.5 \qquad z_{Kacie} = \frac{43-50}{5} = -1.4$$

Kate scored 1.5 standard deviations below the mean, worse than Kacie's 1.4 standard deviations below mean.

5. (22pts) The rainfall for the month of May at a certain location is normally distributed with mean 10.4 inches and standard deviation 2.1 inches. Draw a picture showing which area you are computing as you answer:
a) What percentage of Mays has rainfall between 9 and 12 inches?
b) What percentage of Mays has rainfall greater than 13 inches?
c) What is the percentile of May rainfall of 8.5in? What does this mean?
d) What is the probability that in a random May the rainfall is under 7 inches?

a)

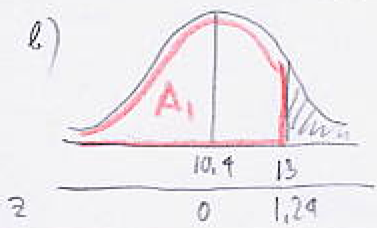


$\frac{12-10.4}{2.1} = 0.76$

$\frac{9-10.4}{2.1} = -0.67$

$A(-0.67 \le z \le 0.76) = A_2 - A_1 = 0.7764 - 0.2514$
$= \boxed{0.525 \ (52.5\%)}$

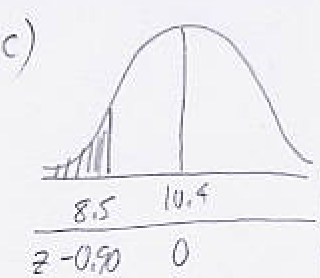
b)



$\frac{13-10.4}{2.1} = 1.24$

$A(z \ge 1.24) = 1 - A_1$
$= 1 - 0.8925 = \boxed{0.1075 \ (10.75\%)}$

c)

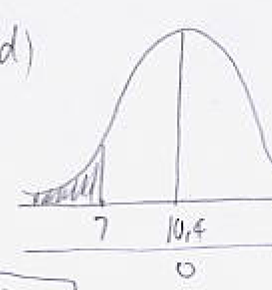


$\frac{8.5-10.4}{2.1} = -0.90$

$A(z \le -0.90) = 0.1841$

$\boxed{\text{18.41\% of Mays have rainfall under 8.5in}}$

d)



$\frac{7-10.4}{2.1} = -1.62$

$A(z \le -1.62)$

$\boxed{= 0.0526 \text{ which is also the probability}}$