1. (12pts) On your commute to class you encounter a red or green traffic light that is red 37% of the time. What is the probability that you
a) encounter red on every day of two days of going to class?
b) encounter green on every day of three days?
c) encounter green on at least one of four days?

a) $P(\text{red 1st day AND red 2nd day}) = P(\text{red 1st day}) \cdot P(\text{red 2nd day})$
$$= 0.37 \cdot 0.37 = 0.1369$$

b) $P(\text{green 1st day AND green 2nd day AND green 3rd day})$
$$= P(\text{green 1st day}) \cdot P(\text{green 2nd day}) \cdot P(\text{green 3rd day})$$
$$= 0.63 \cdot 0.63 \cdot 0.63 = 0.63^3 = 0.250047$$

c) $P(\text{green on at least one of four days}) = P(\text{NOT (red on all four days)})$
$$= 1 - P(\text{red on all four days}) = 1 - 0.37^4 = 1 - 0.01874161 = 0.98125839$$

2. (14pts) A bag contains 13 chestnuts, 10 pecans and 5 walnuts. Three nuts are drawn without replacement. What is the probability that: 28 nuts
a) the second is a pecan, given that the first one is a walnut?
b) all three are chestnuts?
c) first is a chestnut, second is a pecan?
d) at least one is a walnut?

a) $P\left(\begin{smallmatrix}2nd \\ pecan\end{smallmatrix} \middle| \begin{smallmatrix}1st \\ walnut\end{smallmatrix}\right) = \dfrac{10}{27}$

b) $P(\text{1st chestnut AND 2nd chestnut AND 3rd chestnut})$
$$= P(\text{1st chestnut}) \cdot P(\text{2nd chestnut} \mid \text{1st chestnut}) \cdot P(\text{3rd chestnut} \mid \text{1st AND 2nd chestnut})$$
$$= \frac{13}{14\,28} \cdot \frac{12}{27\,9} \cdot \frac{11}{26\,13} = \frac{11}{126} = 0.0873016$$

c) $P(\text{1st chestnut AND 2nd pecan}) = P(\text{1st chestnut}) \cdot P(\text{2nd pecan} \mid \text{1st chestnut})$
$$= \frac{13}{14\,28} \cdot \frac{10^5}{27} = \frac{65}{378} = 0.171958$$

d) $P(\text{at least one is walnut}) = 1 - P(\text{none is a walnut})$
$$= 1 - P\left(\begin{smallmatrix}1st \\ not\ walnut\end{smallmatrix}\right) \cdot P\left(\begin{smallmatrix}2nd \\ not\ walnut\end{smallmatrix} \middle| \begin{smallmatrix}1st \\ not\ walnut\end{smallmatrix}\right) \cdot P(3rd\ not\ walnut \mid 1st\ \&\ 2nd\ not\ walnut)$$
$$= 1 - \frac{23}{2\,14\,28} \cdot \frac{22}{9\,27} \cdot \frac{21}{26} = 1 - \frac{253}{468} = \frac{215}{468} = 0.459402$$

3. (10pts) The table shows the make-up of an automobile dealer's lot by brand and type of vehicle. What is the probability that a random vehicle:
a) is a Ford?
b) is an SUV?
c) is a Lincoln SUV?
d) is an SUV, given it is a Lincoln?
e) is a Ford, given it is a car?

Type	Ford	Lincoln	Total
Car	35	18	53
SUV	32	20	52
Truck	25	3	28
Total	92	41	133

a) $\frac{92}{133} = 0.691729$
b) $\frac{52}{133} = 0.390977$
c) $\frac{20}{133} = 0.150376$
d) $\frac{20}{41} = 0.487805$
e) $\frac{35}{53} = 0.660377$

4. (10pts) A multiple choice test has 5 answers on every question, two of which are correct. You are to select only one answer, and you get 4 points for a correct answer, 0 points for not attempting a question, and 3 points are subtracted for an incorrect answer.
a) What is the expected value of a random guess?
b) If you can rule out one answer as incorrect, what is the expected value of a random guess?
c) If you can always rule out one answer as incorrect and randomly choose an answer among the remaining four, how many points can you expect to have on a 30-question test?

a) Outcomes: $\quad 4 \quad (-3)$

Prob: $\quad \frac{2}{5} \quad \frac{3}{5}$

$E = \frac{2}{5} \cdot 4 + \frac{3}{5}(-3) = \frac{8}{5} - \frac{9}{5} = -\frac{1}{5}$

b) Outcomes $\quad 4 \quad -3$

Prob: $\quad \frac{2}{4} \quad \frac{2}{4}$

$E = \frac{1}{2} \cdot 4 + \frac{1}{2}(-3) = \frac{4-3}{2} = \frac{1}{2}$

c) $30 \cdot \frac{1}{2} = 15$ points

5. (14pts) Dogs Abe, Bo and Charlie compete at a racetrack. Abe wins 36% of races, Bo wins 23% of races, Charlie wins 30% of races, and on 11% of races the dogs lose interest and don't finish. A game of chance is set up as follows: A player pays $10 and collects $20 if Bo wins, $4 if Abe wins, $12 if Charlie wins, and nothing if the dogs lose interest.
a) Find the expected value of this game.
b) If you play this game 40 times, how much do you expect to win or lose?
c) What is the fair price of this game?

a) Outcome (net won):
	A	B	C	None
	-6	10	2	-10
Prob:	0.36	0.23	0.30	0.11

$E = 0.36 \cdot (-6) + 0.23 \cdot 10 + 0.3 \cdot 2 + 0.11(-10)$

$= -0.36$

b) $-0.36 \cdot 40 = -14.40$

May expect to lose $14.40

c) $-0.36 + 10 = \$9.64$