Mathematical Concepts — Joysheet 3
MAT 117, Fall 2012 — D. Ivanšić

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Show all your work!

1. (8pts) Steve would like to have \$3000 for a nice dinner table. How much should he deposit now in an account bearing 3.15%, compounded daily, in order to have the desired amount in two and a half years? How much of the \$3000 came from interest?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$3000 = P\left(1+\frac{0.0315}{365}\right)^{365\cdot 2.5}$

$3000 = P \cdot 1.0819\ldots$

$P = \frac{3000}{1.0819\ldots} = 2772.82$

$3000 - 2772.82 = 227.18$ interest

2. (6pts) Bank of Elephant is offering a 1.35% interest rate on an account that is compounded quarterly, while Donkey Bank has an account at 1.34%, compounded monthly. Which account is the better deal?

$Y = \left(1+\frac{r}{n}\right)^{n} - 1$

Elephant: $Y = \left(1+\frac{0.0135}{4}\right)^{4} - 1 = 0.0135685$, 1.3568% ← a better deal

Donkey: $Y = \left(1+\frac{0.0134}{12}\right)^{12} - 1 = 0.0134826$ 1.34826%

3. (10pts) An investment you are considering is expected to grow at 15%, compounded annually. How long until your investment triples?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$3 = 1\cdot\left(1+\frac{0.15}{1}\right)^{1\cdot t}$

$3 = 1.15^{t} \quad | \log$

$\log 3 = \log 1.15^{t}$

$\log 3 = t \log 1.15$

$t = \frac{\log 3}{\log 1.15} = 7.860597$

About 7.86 years

4. (10pts) To save for a pool, the Hwangs deposit \$1500 every quarter into an account bearing 2.25% interest, compounded quarterly.
a) How much do they have in the account in 3 years?
b) How much did they earn in interest over these 3 years?

a) $A = P\dfrac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$

$A = 1500\dfrac{\left(1+\frac{0.0225}{4}\right)^{4\cdot 3}-1}{\frac{0.0225}{4}}$

$A = 1500 \cdot 12.378\ldots$

$A = 18,567.45$

b) Total deposits $1500 \cdot 4 \cdot 3 = 18000$

Interest $= 18,567.45 - 18,000 = 567.45$

5. (14pts) At the time of little Jason's birth, his parents decided to save some money for his college. They deposit \$4000 immediately into an account bearing 5.2% interest, compounded monthly, and make no further deposits for a while. Then, when Jason went to public kindergarten at age 5, they did not have to pay for day care any more, so they set aside \$350 every month in the same account.
a) How much is in the account when Jason is 18?
b) How much of it was from deposits, and how much from interest?

a) View as two accounts, one with a \$4000 deposit for 18 years and another, with 13 years of \$350 monthly payments. Then combine.

$A_1 = 4000\left(1+\frac{0.052}{12}\right)^{12\cdot 18}$

$= 4000 \cdot 2.54\ldots$

$= 10,178.44$

$A_2 = 350 \cdot \dfrac{\left(1+\frac{0.052}{12}\right)^{12\cdot 13}-1}{\frac{0.052}{12}}$

$= 350 \cdot 222.26\ldots$

$= 77,791.18$

Total: $10,178.44 + 77,791.18$

$= 87,969.62$

b) Total deposits $= 4000 + 350 \cdot 12 \cdot 13 = 58,600$

From interest:

$87969.62 - 58,600$

$= 29,369.62$

6. (12pts) Mikayla would like to save \$2,500 for a fancy refrigerator. If she can set aside \$125 every month into an account bearing 4.56%, compounded monthly, how long will it take her to save the desired amount?

$A = P\dfrac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$

$2500 = 125 \cdot \dfrac{\left(1+\frac{0.0456}{12}\right)^{12t}-1}{\frac{0.0456}{12}}$ $\quad \div 125$

$20 = \dfrac{1.0038^{12t}-1}{0.0038}$ $\quad \cdot 0.0038$

$0.076 = 1.0038^{12t} - 1$ $\quad +1$

$1.076 = 1.0038^{12t}$ $\quad \log$

$\log 1.076 = \log 1.0038^{12t}$

$\log 1.076 = 12t \log 1.0038$

$t = \dfrac{\log 1.076}{12 \log 1.0038} = 1.609420$ years

$\approx$ About 1.61 years