

5. (14pts) In 2011, married couple Jack and Jill, who have two children, filed income taxes jointly. Their total income was \$125,000, they deposited \$10,000 into a retirement account, paid \$8,200 in mortgage interest, \$2,700 in property taxes, \$5,200 in state income taxes and donated \$1,350 to charity. Use the table below to first determine Jack and Jill's taxable income (don't forget the exemptions) and then find the tax on this income.

Income bracket	Tax rate
up to \$17,000	10%
\$17,000–\$69,000	15%
\$69,000–\$139,350	25%
\$139,350–\$212,300	28%
\$212,300–\$379,150	33%
more than \$379,150	35%
exemption per person	\$3,700
standard deduction	\$11,600

6. (14pts) You would like to save up for trip to Brazil.

- How much should you deposit every week into an account with 2.75% interest, compounded weekly, in order to have \$4,000 in two years?
- How much of the final amount is from deposits and how much from interest?

7. (32pts) Interest rates for home mortgages are at their historic lows, making it a good time to buy a home. Suppose your friend takes out a 30-year loan for \$180,000 at 3.5% compounded monthly.

a) What is her monthly payment on the loan?

b) What are her total payments over the course of the loan? How much of this amount is for interest?

c) How much of her first payment goes toward interest, and how much towards the principal?

d) How much does she owe after 18 years?

8. (12pts) If you save for retirement by depositing \$300 every month into an account bearing 8.22% interest, compounded monthly, how long will it take until you have \$200,000 in the account?

Bonus. (10pts) A 20-year \$100,000 mortgage has a monthly payment of \$567.20 at interest rate 3.25%, compounded monthly (you don't need to verify this). Banks allow you to pay more than the monthly rate in an effort to pay off the loan early. If the borrower makes a monthly payment of \$700, how long will it take them to pay off this loan? (*Hint: use only the loan formula.*)

Mathematical Concepts — Exam 2
MAT 117, Fall 2012 — D. Ivanišić

Name: _____
Show all your work!

$$\frac{a}{b} = \frac{P(E)}{1-P(E)} \quad P(E) = \frac{a}{a+b} \text{ where odds in favor of } E \text{ are } a : b \quad P(B | A) = \frac{n(A \text{ and } B)}{n(A)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) = P(A) + P(B) \text{ (if } A \text{ and } B \text{ are mutually exclusive)}$$

$$P(A \text{ and } B) = P(A) \cdot P(B | A) \quad P(A \text{ and } B) = P(A) \cdot P(B) \text{ if } A \text{ and } B \text{ are independent}$$

$$E = P_1 \cdot A_1 + P_2 \cdot A_2 + \cdots + P_n \cdot A_n$$

1. (6pts) In some country license plates of cars consist of three letters and three digits. The middle letter must be a vowel (A, E, I, O, U), while the first letter must be a consonant. The first digit may not be zero or nine, and the three-digit number has to be even. How many different license plates can be made under these rules?

2. (6pts) A student may get a grade of A, B, C, D, E, I in a course. In a class of ten students, how many possibilities are there for the grade sheet of the entire class?

3. (14pts) The table shows the outcomes of car accidents in Florida for a recent year. Write the following probabilities as fractions (that is, no need to write the decimal representation): the probability that a random driver in an accident

- survived?
- did not wear a seat belt?
- wore a seat belt and died?
- died, given that they wore a seat belt?
- survived, given that they did not wear a seat belt?
- wore a seat belt, given that they survived?

	Wore Seat Belt	No Seat belt	Total
Driver Survived	412,300	162,500	
Driver Died	500	1600	
Total			

4. (18pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

Event	probability	odds against	odds in favor
a) Rolling 1 or 3 on a single roll of a die			
b) Drawing a queen or a king from a deck of cards			
c) Getting exactly two heads on three coin tosses			
d) Getting sum divisible by 5 on a roll of two dice			
e) One number odd and one even on a roll of two dice			

5. (10pts) A state has 300 numbered highways. Among those, 78 pass by a lake along their route, 123 have a mile-or-more long segment running through a forest, and 36 have both scenic features. If a numbered highway is selected at random, what is the probability it

- a) passes by a lake along its route or has a mile-or-more segment running through a forest?
- b) doesn't have either scenic feature?

6. (14pts) Players A and B play the following game: each puts \$1 into a bowl and a die is rolled. If 1 or 5 comes up, A gets \$1.72 from the bowl and B gets the remaining \$0.28. If 2, 3 or 6 come up, A gets \$0.54 from the bowl and B gets the remaining \$1.46. If 4 comes up, both players get \$1 back.

a) Compute the expected value of this game from player A's perspective.

b) Compute the expected value of this game from player B's perspective.

c) Which player would you rather be? How much would your preferred player expect to win if they played 250 games?

7. (14pts) Ambitious Dave discovers that he succeeds in 2 projects out of every 5 he embarks on. Assuming that successes on different projects are independent of one another, what is the probability that, working on

a) three projects, Dave succeeds in all of them?

b) two projects, Dave fails on both?

c) five projects, Dave succeeds on at least one?

8. (18pts) A bargain bin at a bookstore contains 13 crime, 10 romance and 15 fantasy novels. If you pick two books at random from the bin, what is the probability that:
- a) both are fantasy novels?
 - b) neither is a crime novel?
 - c) one is a crime novel and the other is a romance novel?

Bonus. (10pts) Lenka likes to exercise on Wednesday and on Saturday but doesn't always find the time or energy to do it. She exercises on a Wednesday 45% of the time. If she exercises on a Wednesday, then she exercises on the following Saturday 35% of the time. If she doesn't exercise on a Wednesday, then she exercises on the following Saturday 80% of the time. In a random week, what is the probability that she exercises on exactly one of Wednesday or Saturday?

Final answers should have accuracy to 6 decimal places (or 4 decimal places for table-derived answers). Show some work how the mean and standard deviation are computed. *Giving only the answer will bring you few points.*

$$\begin{aligned} \text{midrange} &= \frac{\text{lowest value} + \text{highest value}}{2} & \text{range} &= \text{highest value} - \text{lowest value} \\ \bar{x} &= \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum_i x_i}{n} = \frac{\sum_i f_i x_i}{n} & Z &= \frac{X - \bar{x}}{s} \\ s &= \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n - 1}} = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n - 1}} = \sqrt{\frac{\sum_i f_i (x_i - \bar{x})^2}{n - 1}} \end{aligned}$$

1. (18pts) Over the course of two weeks (workdays only) a first-grader counts the number of toys left on the floor after recess. She gets the numbers below.

- a) Find the midrange.
- b) Find the median. 3, 2, 6, 4, 5, 5, 3, 1, 3, 4
- c) Find the mean.
- d) Find the range.
- e) Find the standard deviation.

2. (8pts) A city is considering whether to renovate and expand their existing airport, to be funded by taxpayers. Comment on whether each of the following methods will produce a good random sample of the city's population:

- a Surveying random travelers at the city's bus station.
- b Picking random people from the city's phone book and surveying them.
- c Surveying random employees of the existing airport.
- d Surveying random patrons of an upscale mall.

3. (25pts) A repair shop counts how many vehicles come in each day for an oil change. The data is in the table below (it shows that 2 vehicles came on 5 days, 3 vehicles came on 13 days, etc.)

- a) Draw a histogram for the data.
- b) Find the mode number of daily oil changes.
- c) Find the median number of daily oil changes.
- d) Find the mean number of daily oil changes.
- e) Find the standard deviation.

Oil changes	Frequency (days)
2	5
3	13
4	17
5	11
6	3

4. (6pts) 205 lb Rodolfo is from Brazil, where the weight of men his age is normally distributed with mean 175 lbs and standard deviation 18 lbs. 188 lb Kiran is from India, where the weight of men his age is normally distributed with mean 164 lbs and standard deviation 13 lbs. Use z -scores to determine who is more overweight relative to the populations of their respective countries.

7. (17pts) When elevated to angle 45° , a gun can shoot a shell about 18 kilometers. Actually, tests with many firings of identical shells resulted in a normal distribution of distances with mean 18 km and standard deviation 0.75 km. Draw a picture showing which area you are computing as you answer:

- a) What percentage of shells fall farther than 19km away?
- b) What percentage of shells fall between 17.5 and 18.5 kilometers away?

Bonus. (10pts) Weights of watermelons from a certain field are normally distributed. If it is known that 25% of watermelons have weight under 9 lbs and 75% of watermelons have weight under 12.5 lbs find the mean weight of watermelons and the standard deviation.

(Hint: a part of this problem is the inverse of what we usually do: an area is given and we have to find the z-score. Once you have z-scores, the standard deviation is not far behind.)

Mathematical Concepts — Exam 4
MAT 117, Fall 2012 — D. Ivanišić

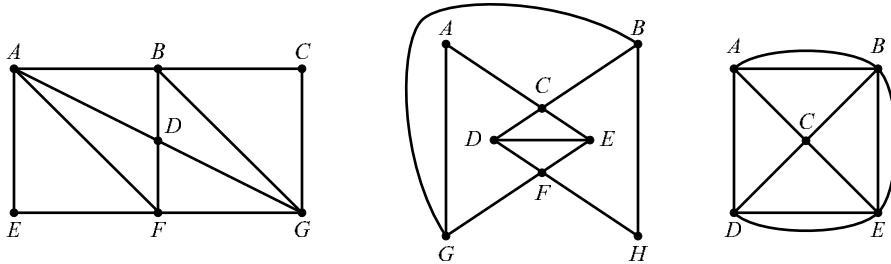
Name: _____
Show all your work!

1. (30pts) A community college for witches is choosing the principal ingredient for its homecoming brew. The finalists are: eye of newt, toe of frog, wool of bat and tongue of dog. The preferences rankings of these ingredients broke down into the following percentages.

Votes	17	16	12	20	10	17	8
1st	EN	EN	TD	TF	TF	WB	WB
2nd	TF	WB	EN	EN	WB	TD	TF
3rd	WB	TD	TF	WB	TD	TF	EN
4th	TD	TF	WB	TD	EN	EN	TD

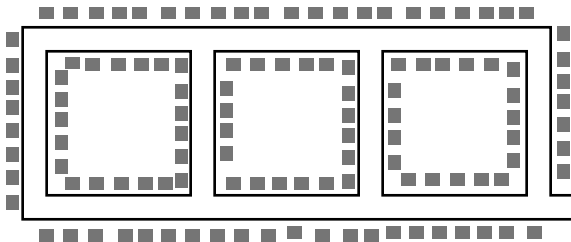
- Which choice wins the vote in a plurality election?
- Which choice wins the vote in a plurality election with elimination?
- Which choice is the pairwise comparison winner?
- Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

2. (17pts) Determine whether each of the following graphs has an Euler path or an Euler circuit. If it does, find it and state the order in which the vertices are visited, if not, explain why not.

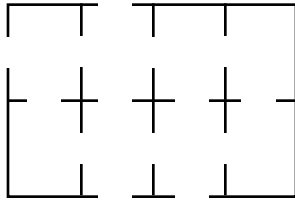


3. (13pts) A mail carrier has to deliver mail to the neighborhood shown in the picture by parking at a corner and walking around the neighborhood. Houses are on both sides of the street, and the mail carrier always walks one row of houses on one side of the street at a time.

- Draw a graph that models the neighborhood.
- Can the mail carrier deliver the mail to every house in the neighborhood without walking by any row of houses twice and return to the starting point? If so, display the route.



4. (14pts) Below is the floor plan of an office building, with doors joining rooms indicated.
- Represent the floor plan as a graph (rooms are vertices, don't forget to include an "outside").
 - Use the graph to determine if it is possible to walk around the office building, passing through every door exactly once. If it is, draw the route.
 - Is it possible to do the same as in b), and start and finish in the same room?



5. (6pts) A salesman has to visit 10 cities. How many different routes are possible (that is, orders of visitation of cities), if he starts and ends in a specified city?

6. (20pts) A tourist would like to visit the German cities Berlin, Frankfurt, Hamburg and Munich, while trying to minimize the distance traveled. The table below has the distances between the cities (in kilometers, of course!).

a) Draw a weighted graph that corresponds to this problem.

b) Use the brute force method to find the route that minimizes the distance traveled. First list all the possible orders of visits with Berlin as the starting city.

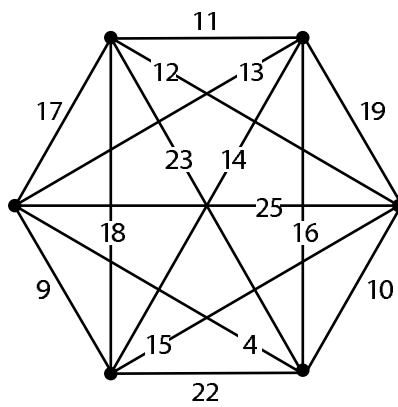
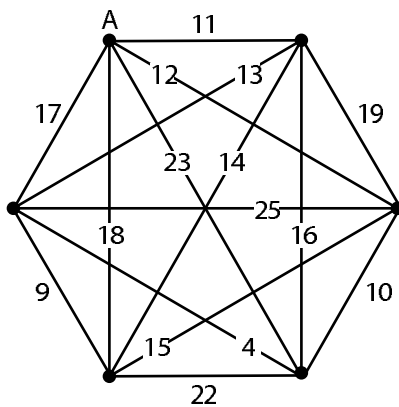
c) Use the nearest neighbor algorithm to find an approximate solution to the problem. Is it the same as in c)?

	B	F	H
F	547		
H	289	492	
M	586	381	776

Bonus. (10pts) Find an approximate solution to the traveling salesman problem. Show the weight of the found circuits. Use (one on each picture)

a) the nearest neighbor algorithm starting at A.

b) the greedy algorithm.



$$I = Prt \quad A = P(1 + rt) \quad A = P \left(1 + \frac{r}{n}\right)^{nt} \quad A = P \frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}}$$

$$P = PMT \frac{1 - \left(1 + \frac{r}{n}\right)^{-nt}}{\frac{r}{n}} \quad Y = \left(1 + \frac{r}{n}\right)^n - 1$$

$$\frac{a}{b} = \frac{P(E)}{1 - P(E)} \quad P(E) = \frac{a}{a+b} \text{ where odds in favor of } E \text{ are } a : b \quad P(B | A) = \frac{n(A \text{ and } B)}{n(A)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) = P(A) + P(B) \text{ (if } A \text{ and } B \text{ are mutually exclusive)}$$

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$$\text{midrange} = \frac{\text{lowest value} + \text{highest value}}{2} \quad \text{range} = \text{highest value} - \text{lowest value}$$

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\sum_i x_i}{n} = \frac{\sum_i f_i x_i}{n} \quad Z = \frac{X - \bar{x}}{s}$$

$$s = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n - 1}} = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n - 1}} = \sqrt{\frac{\sum_i f_i (x_i - \bar{x})^2}{n - 1}}$$

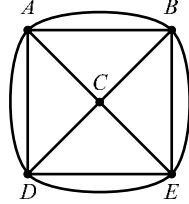
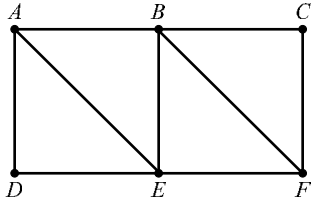
$$E = P_1 \cdot A_1 + P_2 \cdot A_2 + \dots + P_n \cdot A_n$$

1. (24pts) A company is deciding where to take its employees for a “team-building” retreat. The choices are a cruise, Disney World and rafting. A poll of its middle managers results in the following preference rankings.

- Which choice wins the vote in a plurality election?
 - Which choice wins the vote in a plurality election with elimination?
 - Which choice is the pairwise comparison winner?
 - Which choice is the winner using Borda’s method?
- Perform the check on the sum of Borda points.

Votes:	6	2	5	2	3	6
1st	C	C	D	D	R	R
2nd	D	R	C	R	C	D
3rd	R	D	R	C	D	C

2. (12pts) Determine whether each of the following graphs has an Euler path or an Euler circuit. If it does, find it and state the order in which the vertices are visited, if not, explain why not.



3. (25pts) Over one month, a city public transportation employee counts how many of its buses break down in a day and comes up with the following data.

- Draw a histogram for the data.
- Find the mode number of buses broken down.
- Find the median number of buses broken down.
- Find the mean number of buses broken down.
- Find the standard deviation.

Buses broken	Frequency (days)
0	3
1	11
2	9
3	5
4	2

4. (12pts) Psychologists administer a certain skill test and measure the time it takes subjects to complete it. They find that the times for completion are normally distributed with mean 17 seconds and standard deviation 1.5 seconds. Draw a picture showing which area you are computing as you answer:

- a) What percentage of subjects completes the test in less than 16 seconds?
- b) What percentage of subjects completes the test in between 18 and 19 seconds?

5. (12pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

Event	probability	odds against	odds in favor
a) Rolling sum 5 on a roll of two dice			
b) Drawing a king or a red ace from a deck of cards			
c) Getting more heads than tails on three coin tosses			

6. (6pts) Among 57 towns surveyed, 23 had a parking garage downtown and 31 had metered parking downtown, while 46 had a parking garage or metered parking downtown. If a random town is selected, what is the probability that it has both a parking garage and metered parking downtown?

7. (12pts) The following game of chance is offered to you. The cost to play is \$1 and you roll a die. If you roll a 3, you win \$3, if you roll a 4, you win \$2, and you win nothing in all other cases.

a) Compute the expected value of this game.

b) How much would you expect to win or lose if you played 60 games?

8. (10pts) In one year, there is a 7% chance of having a devastating hurricane in Atlantic City, NJ. Assuming that the appearance of hurricanes in different years are independent events, what is the probability that Atlantic City has a devastating hurricane

a) two years in a row?

b) at least once in three consecutive years?

9. (5pts) A nice pair of pants costs \$90. If purchased in Texas, where sales tax is 8.25%, what is the total cost of the pants?

10. (10pts) You borrowed \$2,000 from a bank and repaid it with \$2120 after nine months. What was the interest rate on this loan?

11. (8pts) Rodrigo deposited \$2800 in an account with 3.25% interest, compounded quarterly. How much is in the account in three years?

12. (14pts) When she bought a home, Sandy took out a 25-year \$125,000 mortgage at 3.25% compounded monthly.

a) What is her monthly payment on the loan?

b) What are her total payments over the course of the loan? How much of this amount is for interest?

Bonus. (15pts) A mail carrier has to deliver mail to the neighborhood shown in the picture by parking at a corner and walking around the neighborhood. Houses are on both sides of the street, and the mail carrier always walks one row of houses on one side of the street at a time.

a) Draw a graph that models the neighborhood.

b) Can the mail carrier deliver the mail to every house in the neighborhood without walking by any row of houses twice and return to the starting point? If so, display the route.

