Mathematical Concepts - Exam 2
MAT 117, Spring 2012 - D. Ivanšić

Name:
$\frac{a}{b}=\frac{(E)}{1-P(E)} \quad P(E)=\frac{a}{a+b}$ where odds in favor of $E$ are $a: b \quad P(B \mid A)=\frac{n(A \text { and } B)}{n(A)}$
$P(A$ or $B)=P(A)+P(B)-P(A$ and $B)$
$P(A$ or $B)=P(A)+P(B)$ (if $A$ and $B$ are mutually exclusive)
$P(A$ and $B)=P(A) \cdot P(B \mid A) \quad P(A$ and $B)=P(A) \cdot P(B)$ if $A$ and $B$ are independent
$E=P_{1} \cdot A_{1}+P_{2} \cdot A_{2}+\cdots+P_{n} \cdot A_{n}$

1. (6pts) In a restaurant, there are 17 choices for appetizers, 43 for the main dish and 13 for dessert. Given these choices, how many different three-course meals could you have?
2. (6pts) A die is rolled four times. How many different outcomes does this experiment have?
3. (14pts) The table shows the pattern of mammography results and breast cancer rates among a number of U.S. women ages 40-50. Assuming the numbers are representative for the general population, what is the probability that a random U.S. woman between the ages of 40 and 50 :
a) has breast cancer?
b) had a positive mammogram?
c) does not have breast cancer and had a positive mammogram?
d) has breast cancer, given her mammogram is positive?
e) doesn't have breast cancer, given her mammogram is negative?
f) has a negative mammogram, given that she has breast cancer?

|  | Cancer | No Cancer | Total |
| :---: | ---: | ---: | ---: |
| Positive mammo. | 720 | 6,944 |  |
| Negative mammo. | 80 | 92,256 |  |
| Total |  |  |  |

4. (18pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

|  | Event | probability | odds against | odds in favor |
| :---: | :---: | :---: | :---: | :---: |
| a) | Rolling a 4 on a single roll of a die |  |  |  |
| b) | Drawing a red face card from a deck of cards |  |  |  |
| c) | Getting at least one tail on three coin tosses |  |  |  |
| d) | Getting sum 7 or 8 on a roll of two dice |  |  |  |
| e) | Both numbers odd on a roll of two dice |  |  |  |

5. (14pts) A spinner has 8 equal-size fields, one of which is labeled W , two are labeled I and five are labeled N . A game of chance is set up like this: the player pays $\$ 5$ and spins. Depending on whether the spinner lands on W, I or N the player wins $\$ 15, \$ 7$ or nothing, respectively.
a) Find the expected value of this game.
b) What is the fair price of this game?
c) If a player played this game 100 times, how much would they expect to win or lose?
6. (10pts) In the ice cream bin of a convenience store, $66 \%$ of the products contain vanilla, $47 \%$ contain chocolate, and $31 \%$ have both of those ingredients. If an ice cream product is selected at random, what is the probability that:
a) it contains vanilla or chocolate?
b) it lacks at least one of those ingredients?
7. (14pts) A picky music lover browsing through an online music store finds that, in his opinion, $65 \%$ of the tracks there suck. Choosing tracks randomly, what is the probability that he will select
a) on two tries, both tracks that don't suck?
b) on three tries, all three tracks that suck?
c) on four tries, at least one track that doesn't suck?
8. (18pts) An animal shelter has 7 black kittens, 4 calicos and 5 grey kittens. If you pick two kitties at random, what is the probability that:
a) both are calicos?
b) the first is black and the second is gray?
c) exactly one is grey?

Bonus. (10pts) Two cards are drawn from a deck at random. What is the probability that the first one is a face card and the second one is a club? Hint: this will help you somewhere in the problem: if $B$ and $C$ are mutually exclusive, $P(B$ or $C \mid A)=P(B \mid A)+P(C \mid A)$.

