$I = Prt \quad A = P(1 + rt) \quad A = P\left(1 + \frac{r}{n}\right)^{nt} \quad A = P\frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}}$

$P = PMT\frac{1 - \left(1 + \frac{r}{n}\right)^{-nt}}{\frac{r}{n}} \quad Y = \left(1 + \frac{r}{n}\right)^{n} - 1$

$\frac{a}{b} = \frac{P(E)}{1 - P(E)} \qquad P(E) = \frac{a}{a+b}$ where odds in favor of E are $a : b \qquad P(B\,|\,A) = \frac{n(A \text{ and } B)}{n(A)}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \text{ or } B) = P(A) + P(B)$ (if A and B are mutually exclusive)

$P(A \text{ and } B) = P(A) \cdot P(B\,|\,A) \qquad P(A \text{ and } B) = P(A) \cdot P(B)$ if A and B are independent

$\text{midrange} = \dfrac{\text{lowest value} + \text{highest value}}{2} \qquad \text{range} = \text{highest value} - \text{lowest value}$

$\bar{x} = \dfrac{x_1 + x_2 + \cdots + x_n}{n} = \dfrac{\sum_i x_i}{n} = \dfrac{\sum_i f_i x_i}{n} \qquad Z = \dfrac{X - \bar{x}}{s}$

$s = \sqrt{\dfrac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n - 1}} = \sqrt{\dfrac{\sum_i (x_i - \bar{x})^2}{n - 1}} = \sqrt{\dfrac{\sum_i f_i (x_i - \bar{x})^2}{n - 1}}$

$E = P_1 \cdot A_1 + P_2 \cdot A_2 + \cdots + P_n \cdot A_n$

1. (24pts) A company is deciding where to take its employees for a "team-building" retreat. The choices are a cruise, Disney World and rafting. A poll of its middle managers results in the following preference rankings.

a) Which choice wins the vote in a plurality election?
b) Which choice wins the vote in a plurality election with elimination?
c) Which choice is the pairwise comparison winner?
d) Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

Votes:	6	2	5	2	3	6	= 24 voters
1st	C	C	D	D	R	R	
2nd	D	R	C	R	C	D	
3rd	R	D	R	C	D	C	

a) C 8
   D 7
   R 9 wins.

b) D is eliminated
   C 8 + 5 = 13 wins
   R 9 + 2 = 11

c) C 8+3 = 11
   D 7+6 = 13 wins

   C 8+5 = 13 wins
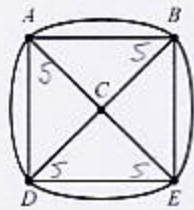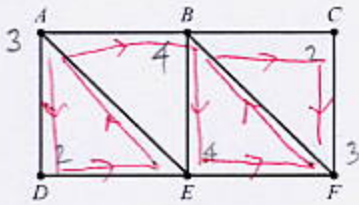   R 9 +2 = 11

   D 7+6 = 13 wins
   R 9+2 = 11

	C	D	R	
pairs	1	2	0	D wins

d) C 3·8 + 2·8 + 1·8 = 48
   D 3·7 + 2·12 + 1·5 = 50
   R 3·9 + 2·9 + 1·11 = 46
   _____
   144

   24·6 = 144, so it checks out
   voters · points per vote

2. (12pts) Determine whether each of the following graphs has an Euler path or an Euler circuit. If it does, find it and state the order in which the vertices are visited, if not, explain why not.

Has exactly two
odd vertices ⇒ has
an Euler path

ADEABEFBCF

Has more than two odd
vertices ⇒ has no Euler path nor Euler circuit.

3. (25pts) Over one month, a city public transportation employee counts how many of its buses break down in a day and comes up with the following data.
a) Draw a histogram for the data.
b) Find the mode number of daily oil changes.
c) Find the median number of daily oil changes.
d) Find the mean number of daily oil changes.
e) Find the standard deviation.

a)

Buses broken	Frequency (days)
0	3
1	11
2	9
3	5
4	2
	30

b) Mode = data item with greatest frequency = 1

c) $0,0,0,1, \ldots, 1, 2, \ldots, 2, 3, \ldots, 3, 4, 4$

3rd   14th 15th 16th

Need 15th, 16th, which are both 2

Median = $\dfrac{2+2}{2}$ = 2

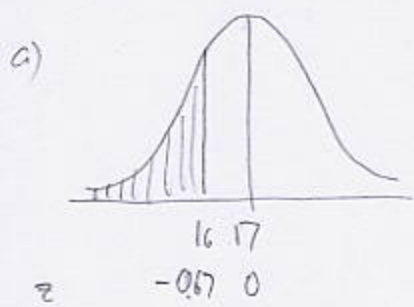d) $\bar{x} = \dfrac{3\cdot 0 + 11\cdot 1 + 9\cdot 2 + 5\cdot 3 + 4\cdot 2}{3+11+9+5+2} = \dfrac{52}{30} = 1.733333$

e) $3(0-1.73\ldots)^2 + 11(1-1.73\ldots)^2 + 9(2-1.73\ldots)^2 + 5\cdot(3-1.73\ldots)^2 + 2(4-1.73\ldots)^2 = 33.866\ldots$

$s = \sqrt{\dfrac{37.866\ldots}{29}} = \sqrt{1.167\ldots} = 1.080655$

4. (12pts) Psychologists administer a certain skill test and measure the time it takes subjects to complete it. They find that the times for completion are normally distributed with mean 17 seconds and standard deviation 1.5 seconds. Draw a picture showing which area you are computing as you answer:
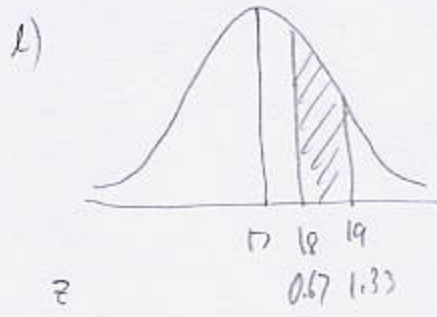a) What percentage of subjects completes the test in less than 16 seconds?
b) What percentage of subjects completes the test in between 18 and 19 seconds?

a)

$$16 \quad 17$$
$$z \qquad -0.67 \quad 0$$

$$z = \frac{16-17}{1.5} = -0.67$$

$$A(z \le -0.67) = 0.2514$$

$$25.14 \%$$

b)

$$17 \quad 18 \quad 19$$
$$z \qquad 0.67 \quad 1.33$$

$$z = \frac{18-17}{1.5} = 0.67 \qquad z = \frac{19-17}{1.5} = 1.33$$

$$A(0.17 \le z \le 1.33) = A_2 - A_1$$
$$= 0.9082 - 0.7486$$
$$= 0.1596$$
$$15.96 \%$$

5. (12pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

	Event	probability	odds against	odds in favor
a)	Rolling sum 5 on a roll of two dice	$\frac{4}{36} = \frac{1}{9}$	$32:4 = 8:1$	$1:8$
b)	Drawing a king or a red ace from a deck of cards	$\frac{6}{52} = \frac{3}{26}$	$46:6 = 23:3$	$3:23$
c)	Getting more heads than tails on three coin tosses	$\frac{4}{8} = \frac{1}{2}$	$4:4 = 1:1$	$1:1$

a) get sums by rolling

1,4, ?
2,3
3,2
4,1

b) 4 keys } 6
   2 red aces }

c) HHH ←
   HHT ← } Have more heads
   HTH ←   than tails
   HTT
   THH ←
   THT
   TTH
   TTT

6. (6pts) Among 57 towns surveyed, 23 had a parking garage downtown and 31 had metered parking downtown, while 46 had a parking garage or metered parking downtown. If a random town is selected, what is the probability that it has both a parking garage and metered parking downtown?

$$P(\text{garage OR metered}) = P(\text{garage}) + P(\text{metered}) - P(\text{garage AND metered})$$

$$\frac{46}{57} = \frac{23}{57} + \frac{31}{57} - P$$

$$\frac{46}{57} = \frac{54}{57} - P$$

$$-\frac{8}{57} = -P$$

$$P(\text{garage AND metered}) = \frac{8}{57}$$

7. (12pts) The following game of chance is offered to you. The cost to play is $1 and you roll a die. If you roll a 3, you win $3, if you roll a 4, you win $2, and you win nothing in all other cases.

a) Compute the expected value of this game.
b) How much would you expect to win or lose if you played 60 games?

a)
	3-1	2-1	
outcomes	2	1	-1
probabilities	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{4}{6}$

$$E = 2\frac{1}{6} + 1 \cdot \frac{1}{6} + (-1) \cdot \frac{4}{6} = \frac{2+1-4}{6} = -\frac{1}{6}$$

b) $$60 \cdot \left(-\frac{1}{6}\right) = -10 \quad \text{Expect to lose } \$10.$$

8. (10pts) In one year, there is a 7% chance of having a devastating hurricane in Atlantic City, NJ. Assuming that the appearance of hurricanes in different years are independent events, what is the probability that Atlantic City has a devastating hurricane

a) two years in a row?

b) at least once in three consecutive years?

a) $P\left(\begin{smallmatrix}Hur.\\in\ 1st\end{smallmatrix} AND \begin{smallmatrix}Hur.\\in\ 2nd\end{smallmatrix}\right) = P\left(\begin{smallmatrix}Hur.\\in\ 1st\end{smallmatrix}\right) \cdot P\left(\begin{smallmatrix}Hur.\\in\ 2nd\end{smallmatrix}\right) = 0.07 \cdot 0.07 = 0.0049$

b) $P\left(at\ least\ once\ in\ 3yrs\right) = 1 - P\left(\begin{smallmatrix}never\\in\ 3\ yrs\end{smallmatrix}\right) = 1 - P\left(\begin{smallmatrix}no\ hur.\\in\ 1st\end{smallmatrix} AND \begin{smallmatrix}no\ hur.\\in\ 2nd\end{smallmatrix} AND \begin{smallmatrix}no\ hur.\\in\ 3rd\end{smallmatrix}\right)$

$= 1 - P\left(\begin{smallmatrix}no\ hur.\\in\ 1st\end{smallmatrix}\right) \cdot P\left(\begin{smallmatrix}no\ hur.\\in\ 2nd\end{smallmatrix}\right) \cdot P\left(\begin{smallmatrix}no\ hur.\\in\ 3rd\end{smallmatrix}\right)$

$= 1 - 0.93^3 = 0.195643$

9. (5pts) A nice pair of pants costs $90. If purchased in Texas, where sales tax is 8.25%, what is the total cost of the pants?

$Tax = 0.0825 \cdot 90 = 7.43$

$Total = 90 + 7.43 = 97.43$

10. (10pts) You borrowed $2,000 from a bank and repaid it with $2120 after nine months. What was the interest rate on this loan?

$A = P(1 + rt)$

$2120 = 2000\left(1 + r \cdot \frac{9}{12}\right) \quad | \div 2000$

$1.06 = 1 + r \cdot \frac{3}{4} \quad | -1$

$0.06 = r \cdot \frac{3}{4} \quad | \cdot \frac{4}{3}$

$r = 0.06 \cdot \frac{4}{3} = 0.08, \quad so \quad 8\%$

11. (8pts) Rodrigo deposited $2800 in an account with 3.25% interest, compounded quarterly. How much is in the account in three years?

$$A = P\left(1+ \frac{r}{u}\right)^{ut} \qquad A = 2800\left(1+ \frac{0.0325}{4}\right)^{4\cdot3} = 2800 \cdot 1.1019\_ = 3085.54$$

12. (14pts) When she bought a home, Sandy took out a 25-year $125,000 mortgage at 3.25% compounded monthly.
a) What is her montly payment on the loan?
b) What are her total payments over the course of the loan? How much of this amount is for interest?

a) $P = PMT \dfrac{1-\left(1+\frac{r}{u}\right)^{-ut}}{\frac{x}{u}}$ 300

$125000 = PMT \cdot \dfrac{1-\left(1+\frac{0.0325}{12}\right)^{-12\cdot25}}{\frac{0.0325}{12}}$

$125\,000 = PMT \cdot 205.20\_$

$PMT = \dfrac{125000}{205.20\_} = 609.15$

b) $609.15 \cdot 12 \cdot 25 = 182{,}745$

$182{,}745 - 125000 = 57{,}745$ in interest

Bonus. (15pts) A mail carrier has to deliver mail to the neighborhood shown in the picture by parking at a corner and walking around the neighborhood. Houses are on both sides of the street, and the mail carrier always walks one row of houses on one side of the street at a time.
a) Draw a graph that models the neighborhood.
b) Can the mail carrier deliver the mail to every house in the neighborhood without walking by any row of houses twice and return to the starting point? If so, display the route.
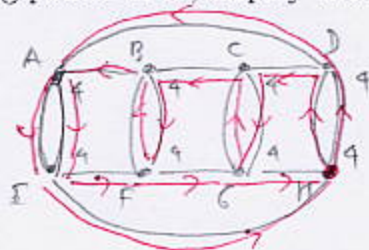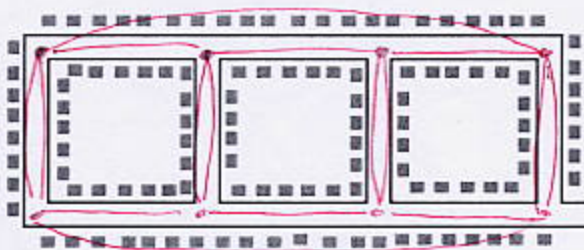
All vertices even ⇒ has an Euler circuit

HDAEHDCGCBFBAEFGH