Mathematical Concepts — Exam 4
MAT 117, Fall 2012 — D. Ivanšić

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Show all your work!

1. (30pts) A community college for witches is choosing the principal ingredient for its homecoming brew. The finalists are: eye of newt, toe of frog, wool of bat and tongue of dog. The preferences rankings of these ingredients broke down into the following percentages.

Votes	17	16	12	20	10	17	8
1st	EN	EN	TD	TF	TF	WB	WB
2nd	TF	WB	EN	EN	WB	TD	TF
3rd	WB	TD	TF	WB	TD	TF	EN
4th	TD	TF	WB	TD	EN	EN	TD

→ 100 altogether

a) Which choice wins the vote in a plurality election?
b) Which choice wins the vote in a plurality election with elimination?
c) Which choice is the pairwise comparison winner?
d) Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

a) EN 33 wins
TD 12
TF 30
WB 25

dropped in round 2

b) EN 33+12=45
TF 30
WB 25

dropped in round 3

EN 33+12 = 45
TF 30+17+8=55 wins

c) EN 33+20+8=61 wins
TD 12+10+17=39

EN 33+12 =45
TF 30+17+8=55 win

EN 33+12+20=65 wins
WB 25+10 =35

TD 12+16+17=45
TF 30+17+8=55 wins

TD 12+ =12
WB 25+17+16+20+10=88 wins

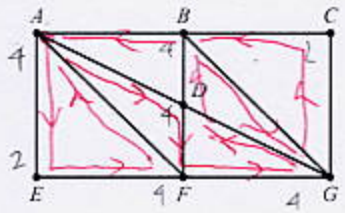
TF 30+17+12 =59 wins
WB 25+16 =41

Points: EN 2, TD 0, TF 3, WB 1
wins

d) EN $4\cdot33 + 3\cdot32 + 2\cdot8 + 1\cdot27 = 271$
TD $4\cdot12 + 3\cdot17 + 2\cdot26 + 1\cdot45 = 196$
TF $4\cdot30 + 3\cdot25 + 2\cdot29 + 1\cdot16 = 269$
WB $4\cdot25 + 3\cdot26 + 2\cdot37 + 1\cdot12 = 264$
1000

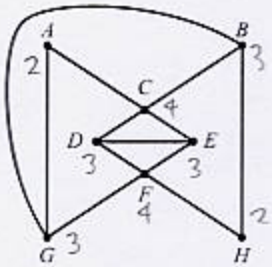
$100 \cdot 10 = 1000$
↑ voters ↑ pts. per voter

2. (17pts) Determine whether each of the following graphs has an Euler path or an Euler circuit. If it does, find it and state the order in which the vertices are visited, if not, explain why not.

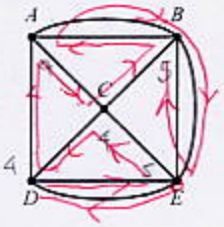


All vertices even
⇒ has Euler circuit

AEFADFGDBGCBA



More than two odd vertices
⇒ has neither Euler path nor circuit



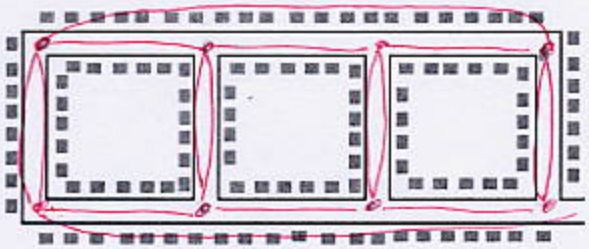
Exactly two odd vertices
⇒ has Euler path (but no circuit)

EDECDACBABEB

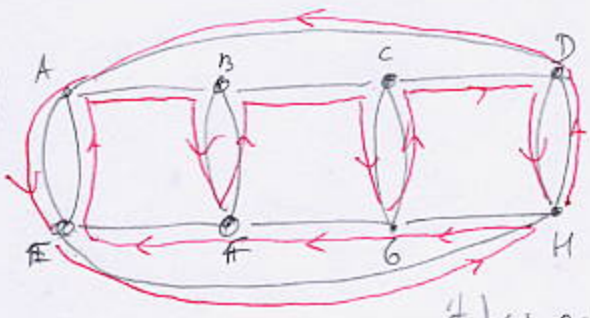
3. (13pts) A mail carrier has to deliver mail to the neighborhood shown in the picture by parking at a corner and walking around the neighborhood. Houses are on both sides of the street, and the mail carrier always walks one row of houses on one side of the street at a time.

a) Draw a graph that models the neighborhood.

b) Can the mail carrier deliver the mail to every house in the neighborhood without walking by any row of houses twice and return to the starting point? If so, display the route.



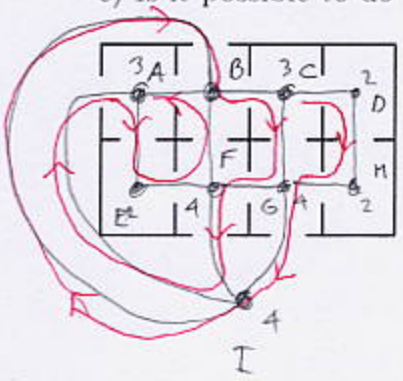
a)



HDAEH GFEABFBCGC DH

b) All vertices even, so it has an Euler circuit, so yes, it is possible

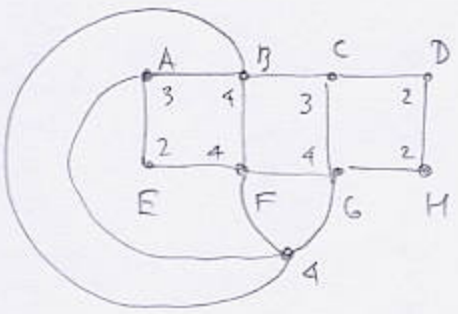
4. (14pts) Below is the floor plan of an office building, with doors joining rooms indicated.
a) Represent the floor plan as a graph (rooms are vertices, don't forget to include an "outside").
b) Use the graph to determine if it is possible to walk around the office building, passing through every door exactly once. If it is, draw the route.
c) Is it possible to do the same as in b), and start and finish in the same room?



b) Graph has exactly two odd vertices, so has an Euler path (starts and ends at odd vertices) but no Euler circuit

c) No, because there is no Euler circuit

C D H G I B C G F I A E F B A



5. (6pts) A salesman has to visit 10 cities. How many different routes are possible (that is, orders of visitation of cities), if he starts and ends in a specified city?

Start _ _ _ _ _ _ _ _ _ end

choices 9 8 7 ... 3 2 1

Number of possible paths: $9 \cdot 8 \cdot 7 \cdots 3 \cdot 2 \cdot 1 = 9! = 362880$

6. (20pts) A tourist would like to visit the German cities Berlin, Frankfurt, Hamburg and Munich, while trying to minimize the distance traveled. The table below has the distances between the cities (in kilometers, of course!).

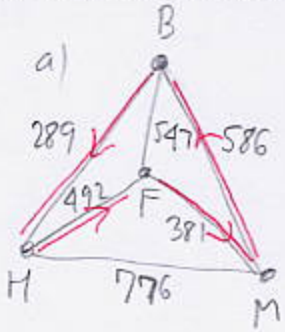
a) Draw a weighted graph that corresponds to this problem.

b) Use the brute force method to find the route that minimizes the distance traveled. First list all the possible orders of visits with Berlin as the starting city.

c) Use the nearest neighbor algorithm to find an approximate solution to the problem. Is it the same as in b)?

	B	F	H
F	547		
H	289	492	
M	586	381	776

a)



b) B F H M B $547+492+776+586 = 2401$

B F M H B $547+381+776+289 = 1993$

B H F M B $289+492+381+586 = 1748$ best

B H M F B $289+776+381+547 = 1993$

B M F H B $586+381+492+289 = 1748$

B M H F B $586+776+492+547 = 2401$

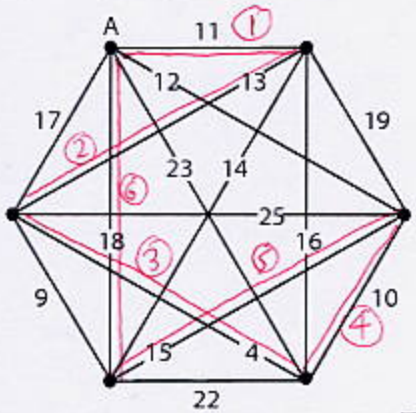
c) Nearest neighbor (see picture in a)

B H F M B – 1748 gives the best route

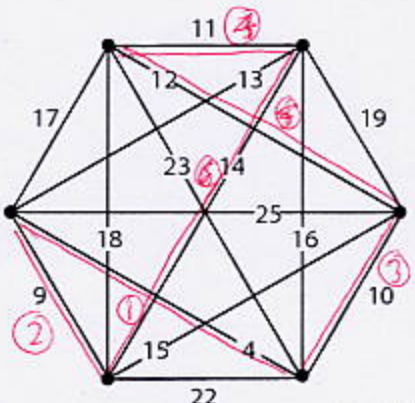
Bonus. (10pts) Find an approximate solution to the traveling salesman problem. Show the weight of the found circuits. Use (one on each picture)

a) the nearest neighbor algorithm starting at A.

b) the greedy algorithm.



$11+13+4+10+15+18 = 71$



$4+9+14+11+12+10 = 60$