**Mathematical Concepts — Exam 3**
**MAT 117, Fall 2012 — D. Ivanšić**

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*Show all your work!*

Final answers should have accuracy to 6 decimal places (or 4 decimal places for table-derived answers). Show some work how the mean and standard deviation are computed. *Giving only the answer will bring you few points.*

$$\text{midrange} = \frac{\text{lowest value} + \text{highest value}}{2} \qquad \text{range} = \text{highest value} - \text{lowest value}$$

$$\overline{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum_i x_i}{n} = \frac{\sum_i f_i x_i}{n} \qquad Z = \frac{X - \overline{x}}{s}$$

$$s = \sqrt{\frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \cdots + (x_n - \overline{x})^2}{n-1}} = \sqrt{\frac{\sum_i (x_i - \overline{x})^2}{n-1}} = \sqrt{\frac{\sum_i f_i (x_i - \overline{x})^2}{n-1}}$$

**1.** (18pts) Over the course of two weeks (workdays only) a first-grader counts the number of toys left on the floor after recess. She gets the numbers below.

a) Find the midrange.
b) Find the median.
c) Find the mean.
d) Find the range.
e) Find the standard deviation.

3, 2, 6, 4, 5, 5, 3, 1, 3, 4

a) $\frac{1+6}{2} = 3.5$

b) 1, 2, 3, 3, 3, 4, 4, 5, 5, 6 (10 items)

Need 5th and 6th

$\frac{3+4}{2} = 3.5$

c) $\bar{x} = \frac{3+2+6+\cdots+1+3+4}{10} = \frac{36}{10} = 3.6$

d) $6 - 1 = 5$

e) $(1-3.6)^2 + (2-3.6)^2 + 3(3-3.6)^2 + 2\cdot(4-3.6)^2 + 2(5-3.6)^2 + (6-3.6)^2$

$= 20.4$

$s = \sqrt{\frac{20.4}{9}} = \sqrt{2.6666\ldots} = 1.505545$

**2.** (8pts) A city is considering whether to renovate and expand their existing airport, to be funded by taxpayers. Comment on whether each of the following methods will produce a good random sample of the city's population:

a Surveying random travelers at the city's bus station.
b Picking random people from the city's phone book and surveying them.
c Surveying random employees of the existing airport.
d Surveying random patrons of an upscale mall.

a) Since people who travel by bus tend to be lower income, so they don't fly, this sample may not be very interested in the project

b) Probably the best method of the four, although it misses people who only have a cell phone

c) Poor sample - this group is naturally tilted in favor of the project

d) Wealthier people tend to fly more, so this group may be biased in favor of the airport.

**3.** (25pts) A repair shop counts how many vehicles come in each day for an oil change. The data is in the table below (it shows that 2 vehicles came on 5 days, 3 vehicles came on 13 days, etc.)

a) Draw a histogram for the data.
b) Find the mode number of daily oil changes.
c) Find the median number of daily oil changes.
d) Find the mean number of daily oil changes.
e) Find the standard deviation.

| Oil changes | Frequency (days) |
|---|---|
| 2 | 5 |
| 3 | 13 |
| 4 | 17 |
| 5 | 11 |
| 6 | 3 |
| | 49 |

a)


b) mode is 4 (data item with highest frequency)

c) 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 6
(5th, 18th, 35th)

Need $\frac{49}{2} = 24.5$, 25th number in list, which is $\boxed{4}$

d) $\bar{x} = \frac{5\cdot 2 + 13\cdot 3 + 17\cdot 4 + 11\cdot 5 + 3\cdot 6}{49} = \frac{190}{49} = 3.877551$

e) $5(2-3.87..)^2 + 13(3-3.87..)^2 + 17(4-3.87..)^2 + 11(5-3.87..)^2 + 3(6-3.87..)^2 = 55.26..$

$s = \sqrt{\frac{56.26..}{48}} = \sqrt{1.151..} = 1.073015$

**4.** (6pts) 205 lb Rodolfo is from Brazil, where the weight of men his age is normally distributed with mean 175 lbs and standard deviation 18 lbs. 188 lb Kiran is from India, where the weight of men his age is normally distributed with mean 164 lbs and standard deviation 13 lbs. Use $z$-scores to determine who is more overweight relative to the populations of their respective countries.

Rodolfo:

$z = \frac{205-175}{18} = 1.666667$

Kiran

$z = \frac{188-169}{13} = 1.846154$

$\approx 1.85$ stand deviations above mean, hence more overweight.

**5.** (14pts) The scores above 40 on exam 2 of our class are shown below.
a) Construct a grouped frequency distribution whose first class is 40–49.
b) Enter a representative value for each interval.
c) Use the representative values to estimate the mean of data. How does it compare to the actual mean of 77? (Do not compute the actual mean.)

95, 70, 93, 91, 94, 97, 61, 66, 69, 75, 101, 102, 59, 47, 80, 44, 87, 84, 81, 102, 67, 53, 83, 62, 73, 42, 104, 95, 70, 99, 60, 47, 73, 86, 86, 71, 80, 76

| Class | Frequency | Representative Value |
|---|---|---|
| 40-49 | 4 | 44.5 = (40+49)/2 |
| 50-59 | 2 | 54.5 |
| 60-69 | 6 | 64.5 |
| 70-79 | 7 | 74.5 |
| 80-89 | 8 | 84.5 |
| 90-99 | 7 | 94.5 |
| 100-109 | 4 | 104.5 |
| | 38 | |

c) $\frac{4 \cdot 44.5 + 2 \cdot 54.5 + \ldots + 7 \cdot 94.5 + 4 \cdot 104.5}{38}$

$= \frac{2951}{38} = 77.657895$

Estimate of mean is pretty close to actual mean.

**6.** (12pts) The weight of baby girls at age 12 months is normally distributed with mean 9.5 kg and standard deviation 1 kg. Use the 68-95-99.7 rule (draw a picture) to find the percentage of twelve-month-old girls whose weight is

a) between 8.5 and 9.5

$\frac{0.68}{2} = 0.34$ (34%)

b) over 8.5

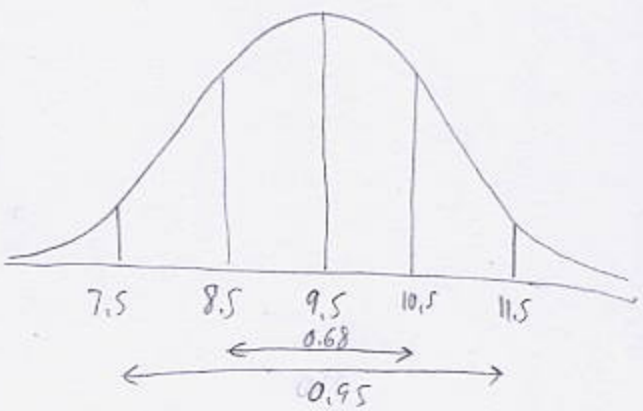
$0.34 + 0.5 = 0.84$ (84%)

c) under 11.5

$0.5 + \frac{0.95}{2} = 0.975$ (97.5%)

d) between 7.5 and 10.5

$\frac{0.95}{2} + \frac{0.68}{2} = 0.815$ (81.5%)

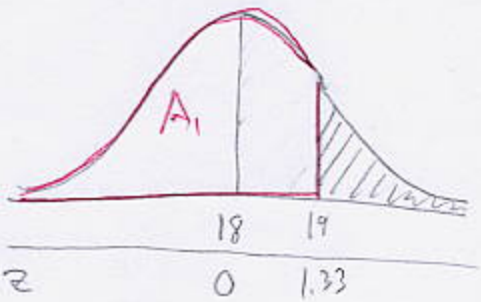


**7.** (17pts) When elevated to angle 45°, a gun can shoot a shell about 18 kilometers. Actually, tests with many firings of identical shells resulted in a normal distribution of distances with mean 18 km and standard deviation 0.75 km. Draw a picture showing which area you are computing as you answer:

a) What percentage of shells fall farther than 19km away?

b) What percentage of shells fall between 17.5 and 18.5 kilometers away?

a)




$\frac{19-18}{0.75} \approx 1.33$

$A(z \geq 1.33) = 1 - A_1 = 1 - 0.9082 = 0.0918$

$(9.18\%)$

b)


$\frac{18.5-18}{0.75} = 0.67$ $\quad \frac{17.5-18}{0.75} = -0.67$

$A(-0.67 \leq z \leq 0.67) = A_2 - A_1 = 0.7486 - 0.2514$

$= 0.4972$

$(49.72\%)$

**Bonus.** (10pts) Weights of watermelons from a certain field are normally distributed. If it is known that 25% of watermelons have weight under 9 lbs and 75% of watermelons have weight under 12.5 lbs find the mean weight of watermelons and the standard deviation.
*(Hint: a part of this problem is the inverse of what we usually do: an area is given and we have to find the z-score. Once you have z-scores, the standard deviation is not far behind.)*


Need z-scores corresponding to areas 0.25 and 0.75. The z-scores with areas closest to 0.25, 0.75 are

$z_1 = -0.67$ and $z_2 = 0.67$

Then $\frac{9 - 10.75}{s} = -0.67$

$-1.75 = -0.675$

$s = \frac{1.75}{0.67} = 2.611940,$

By symmetry, we have

$\bar{x} = \frac{9+12.5}{2} = 10.75$