Mathematical Concepts — Exam 2
MAT 117, Fall 2012 — D. Ivanšić

Name: Saul Ocean

Show all your work!

$\frac{a}{b} = \frac{P(E)}{1-P(E)}$ $\quad P(E) = \frac{a}{a+b}$ where odds in favor of E are $a:b$ $\quad P(B \mid A) = \frac{n(A \text{ and } B)}{n(A)}$
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
$P(A \text{ or } B) = P(A) + P(B)$ (if A and B are mutually exclusive)
$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$ $\quad P(A \text{ and } B) = P(A) \cdot P(B)$ if A and B are independent
$E = P_1 \cdot A_1 + P_2 \cdot A_2 + \cdots + P_n \cdot A_n$

1. (6pts) In some country license plates of cars consist of three letters and three digits. The middle letter must be a vowel (A, E, I, O, U), while the first letter must be a consonant. The first digit may not be zero or nine, and the three-digit number has to be even. How many different license plates can be made under these rules?

not AEIOU	AEIOU	any	not 0,9	any	0,2,4,6,8
↑	↑	↑	↑	↑	↑
21	5	26	8	10	5

choices

$21 \cdot 5 \cdot 26 \cdot 8 \cdot 10 \cdot 5$

$= 1,092,000$

2. (6pts) A student may get a grade of A, B, C, D, E, I in a course. In a class of ten students, how many possibilities are there for the grade sheet of the entire class?

AB
CD
EI

choices: 6 6 — — 6

$\underbrace{6 \cdot \ldots \cdot 6}_{10 \text{ times}} = 6^{10} = 60,466,176$

3. (14pts) The table shows the outcomes of car accidents in Florida for a recent year. Write the following probabilities as fractions (that is, no need to write the decimal representation): the probability that a random driver in an accident

a) survived?
b) did not wear a seat belt?
c) wore a seat belt and died?
d) died, given that they wore a seat belt?
e) survived, given that they did not wear a seat belt?
f) wore a seat belt, given that they survived?

	Wore Seat Belt	No Seat belt	Total
Driver Survived	412,300	162,500	574,800
Driver Died	500	1600	2100
Total	412,800	164,100	576,900

a) $\frac{574,800}{576,900} = \frac{1916}{1923} \approx 0.996360$

b) $\frac{164,100}{576,900} = \frac{547}{1923} \approx 0.284451$

c) $\frac{500}{576,900} = \frac{5}{5769} \approx 0.000867$

d) $\frac{500}{412,800} = \frac{5}{4128} \approx 0.001211$

e) $\frac{162,500}{164,100} = \frac{1625}{1641} \approx 0.990250$

f) $\frac{412,300}{574,800} = \frac{4123}{5748} \approx 0.717293$

4. (18pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

	Event	probability	odds against	odds in favor
a)	Rolling 1 or 3 on a single roll of a die	$\frac{2}{6} = \frac{1}{3}$	4 to 2 = 2 to 1	1 to 2
b)	Drawing a queen or a king from a deck of cards	$\frac{8}{52} = \frac{2}{13}$	11 to 2	2 to 11
c)	Getting exactly two heads on three coin tosses	$\frac{3}{8}$	5 to 3	3 to 5
d)	Getting sum divisible by 5 on a roll of two dice	$\frac{7}{36}$	29 to 7	7 to 29
e)	One number odd and one even on a roll of two dice	$\frac{18}{36}$	18 to 18 = 1 to 1	1 to 1

b) 4 queens & 4 kings in a deck

c) HHH ←
HHT
HTH ←
HTT
THH ←
THT
TTH
TTT

d) div. by 5 = 5 or 10

7 in all: 1,4   2,3   3,2   4,1   4,6   5,5   6,4

e) odd even: 3 3   even odd: 3 3

$3 \cdot 3 + 3 \cdot 3 = 18$

5. (10pts) A state has 300 numbered highways. Among those, 78 pass by a lake along their route, 123 have a mile-or-more long segment running through a forest, and 36 have both scenic features. If a numbered highway is selected at random, what is the probability it
a) passes by a lake along its route or has a mile-or-more segment running through a forest?
b) doesn't have either scenic feature?

a) P(lake OR forest) = P(lake) + P(forest) − P(lake AND forest)

$P(\text{lake OR forest}) = \frac{78}{300} + \frac{123}{300} - \frac{36}{300} = \frac{165}{300} = \frac{55}{100} = 0{,}55$

b) P(doesn't have either lake or forest) = P(NOT (lake OR forest))

$= 1 - P(\text{lake OR forest}) = 1 - \frac{55}{100} = \frac{100-55}{100} = \frac{45}{100} = 0{,}45$

6. (14pts) Players A and B play the following game: each puts \$1 into a bowl and a die is rolled. If 1 or 5 comes up, A gets \$1.72 from the bowl and B gets the remaining \$0.28. If 2, 3 or 6 come up, A gets \$0.54 from the bowl and B gets the remaining \$1.46. If 4 comes up, both players get \$1 back.
a) Compute the expected value of this game from player A's perspective.
b) Compute the expected value of this game from player B's perspective.
c) Which player would you rather be? How much would your preferred player expect to win if they played 250 games?

a) Outcomes (net win): $\underset{(1.72-1)}{0.72}$ $\underset{(0.54-1)}{-0.46}$ 0
Prob. $\frac{2}{6}$ $\frac{3}{6}$ $\frac{1}{6}$

$$E = \frac{2}{6}\cdot 0.72 + \frac{3}{6}(-0.46) + 0 = \frac{1.44 - 1.38}{6} = \frac{0.06}{6} = 0.01$$

b) Outcomes (net win) $\underset{(0.28-1)}{-0.72}$ $\underset{(1.46-1)}{0.46}$ 0
Prob. $\frac{2}{6}$ $\frac{3}{6}$ $\frac{1}{6}$

$$E = \frac{2}{6}\cdot(-0.72) + \frac{3}{6}\cdot(0.46) + 0 = \frac{-1.44 + 1.38}{6} = \frac{-0.06}{6} = -0.01$$

c) A, their expected value is positive (may expect to win)

$250 \cdot 0.01 = 2.50$ A expects to win \$2.50 over 250 games.

7. (14pts) Ambitious Dave discovers that he succeeds in 2 projects out of every 5 he embarks on. Assuming that successes on different projects are independent of one another, what is the probability that, working on
a) three projects, Dave succeeds in all of them?
b) two projects, Dave fails on both?
c) five projects, Dave succeeds on at least one?

a) $P(\text{succeeds on all three}) = P(\text{1st succ. AND 2nd succ. AND 3rd succ.}) = [\text{events are independent}]$

$$= P(\text{1st succ}) \cdot P(\text{2nd succ}) \cdot P(\text{3rd succ.}) = \frac{2}{5}\cdot\frac{2}{5}\cdot\frac{2}{5} = \frac{8}{125} = 0.064$$

b) $P(\text{fails both}) = P(\text{1st fail AND 2nd fail}) = P(\text{1st fail}) \cdot P(\text{2nd fail}) = \frac{3}{5}\cdot\frac{3}{5} = \frac{9}{25} = 0.36$

c) $P(\text{succeeds on at least one of five}) = 1 - P(\text{fails on all five})$

$$= 1 - \frac{3}{5}\cdot\frac{3}{5}\cdot\frac{3}{5}\cdot\frac{3}{5}\cdot\frac{3}{5} = 1 - \left(\frac{3}{5}\right)^5$$

$$= 1 - 0.07776 = 0.92224 = \frac{2882}{3125}$$

$\overbrace{\qquad\qquad\qquad}^{38 \text{ in all}}$

8. (18pts) A bargain bin at a bookstore contains 13 crime, 10 romance and 15 fantasy novels. If you pick two books at random from the bin, what is the probability that:
a) both are fantasy novels?
b) neither is a crime novel?
c) one is a crime novel and the other is a romance novel?

a) $P\left(\begin{matrix}\text{1st}\\\text{fantasy}\end{matrix} \text{ AND } \begin{matrix}\text{2nd}\\\text{Fantasy}\end{matrix}\right) = P\left(\begin{matrix}\text{1st}\\\text{fantasy}\end{matrix}\right) \cdot P\left(\begin{matrix}\text{2nd}\\\text{fantasy}\end{matrix} \,\middle|\, \begin{matrix}\text{1st}\\\text{fantasy}\end{matrix}\right) = \frac{15}{\cancel{38}_{19}} \cdot \frac{\cancel{14}^{7}}{37} = \boxed{\frac{105}{703} \approx 0.149360}$

b) $P\left(\begin{matrix}\text{1st}\\\text{NOT crime}\end{matrix} \text{ AND } \begin{matrix}\text{2nd}\\\text{NOT crime}\end{matrix}\right) = P\left(\begin{matrix}\text{1st}\\\text{NOT crime}\end{matrix}\right) \cdot P\left(\begin{matrix}\text{2nd}\\\text{NOT crime}\end{matrix} \,\middle|\, \begin{matrix}\text{1st}\\\text{NOT crime}\end{matrix}\right)$

$= \frac{25}{19\cancel{38}} \cdot \frac{\cancel{24}^{12}}{37} = \boxed{\frac{300}{703} = 0.426743}$

c) $P(\text{one is crime, one is romance}) = P\left(\underbrace{\left(\begin{matrix}\text{1st}\\\text{crime}\end{matrix} \text{ AND } \begin{matrix}\text{2nd}\\\text{romance}\end{matrix}\right) \text{ OR } \left(\begin{matrix}\text{1st}\\\text{romance}\end{matrix} \text{ AND } \begin{matrix}\text{2nd}\\\text{crime}\end{matrix}\right)}_{\text{mutually exclusive}}\right)$

$= P(\text{1st crime AND 2nd romance}) + P(\text{1st romance AND 2nd crime})$

$= P(\text{1st crime}) \cdot P\left(\begin{matrix}\text{2nd}\\\text{romance}\end{matrix} \,\middle|\, \begin{matrix}\text{1st}\\\text{crime}\end{matrix}\right) + P\left(\begin{matrix}\text{1st}\\\text{romance}\end{matrix}\right) \cdot P\left(\begin{matrix}\text{2nd}\\\text{crime}\end{matrix} \,\middle|\, \begin{matrix}\text{1st}\\\text{romance}\end{matrix}\right) = \frac{13}{38} \cdot \frac{10}{37} + \frac{10}{38} \cdot \frac{13}{37}$

$= \frac{2 \cdot 10 \cdot 13}{38 \cdot 37}$

$\boxed{= \frac{130}{703} = 0.184922}$

Bonus. (10pts) Lenka likes to exercise on Wednesday and on Saturday but doesn't always find the time or energy to do it. She exercises on a Wednesday 45% of the time. If she exercises on a Wednesday, then she exercises on the following Saturday 35% of the time. If she doesn't exercise on a Wednesday, then she exercises on the following Saturday 80% of the time. In a random week, what is the probability that she exercises on exactly one of Wednesday or Saturday?

$P(\text{exactly one of } W, S) = P\left(\underbrace{(W \text{ AND NOT } S) \text{ OR } ((\text{NOT } W) \text{ AND SAT})}_{\text{mutually exclusive}}\right)$

$= P(W \text{ AND } (\text{NOT } S)) + P((\text{NOT } W) \text{ AND } S)$

$= P(W) \cdot P(\text{NOT } S \mid W) + P(\text{NOT } W) \cdot P(S \mid \text{NOT } W)$

$= 0.45 \cdot \underbrace{0.65}_{(1-0.35)} + \quad + 0.55 \cdot 0.8 = 0.2925 + 0.44 = 0.7325$