Mathematical Concepts — Exam 1
MAT 117, Fall 2012 — D. Ivanšić

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Show all your work!

$$I = Prt \quad A = P(1+rt) \quad A = P\left(1+\frac{r}{n}\right)^{nt} \quad A = P\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}} \quad P = PMT\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}} \quad Y = \left(1+\frac{r}{n}\right)^{n}-1$$

1. (5pts) 43 is ~~what~~ 13 percent of what number?

$A = PB$

$43 = 0.13 \cdot B$

$B = \frac{43}{0.13} = 330.769231$

2. (5pts) A new tablet computer costs \$199. If purchased in Kentucky, where sales tax is 6%, what is the total cost of the tablet?

$tax = 0.06 \cdot 199 = 11.94$

$total\ cost = 199 + 11.94 = 210.94$

3. (10pts) You borrowed \$1,300 from a bank at simple interest of 7%. If you repaid the loan with \$1446.38, how long did it take you to repay the loan?

$A = P(1+rt)$

$1446.38 = 1300(1+0.07 \cdot t) \quad | \div 1300$

$1.1126 = 1+0.07t \quad | -1$

$0.1126 = 0.07t \quad | \div 0.07$

$t = \frac{0.1126}{0.07} = 1.608571$ years

4. (8pts) Phil deposited \$1500 in an account with 4.62% interest, compounded quarterly. How much is in the account in five years?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = 1500\left(1+\frac{0.0462}{4}\right)^{4 \cdot 5} = 1500 \cdot 1.25\ldots$

$= 1887.29$

5. (14pts) In 2011, married couple Jack and Jill, who have two children, filed income taxes jointly. Their total income was \$125,000, they deposited \$10,000 into a retirement account, paid \$8,200 in mortgage interest, \$2,700 in property taxes, \$5,200 in state income taxes and donated \$1,350 to charity. Use the table below to first determine Jack and Jill's taxable income (don't forget the exemptions) and then find the tax on this income.

Income bracket	Tax rate
up to \$17,000	10%
\$17,000–\$69,000	15%
\$69,000–\$139,350	25%
\$139,350–\$212,300	28%
\$212,300–\$379,150	33%
more than \$379,150	35%
exemption per person	\$3,700
standard deduction	\$11,600

deductions: 8200
2700
5200
1350
17,450
→ greater than standard deduction, so use that

Taxable income = $125000 - (10,000 + 17,450 + 4 \cdot 3,700) = 125,000 - 42250 = 82,750$

Tax on 82,750 is $0.10 \cdot 17000 + 0.15(69,000 - 17,000) + 0.25 \cdot (82750 - 69000)$

$= 0.10 \cdot 17,000 + 0.15 \cdot 42,000 + 0.25 \cdot 13,750$

$= 1700 + 7800 + 3437.5 = \boxed{12,937.50}$

6. (14pts) You would like to save up for trip to Brazil.
a) How much should you deposit every week into an account with 2.75% interest, compounded weekly, in order to have \$4,000 in two years?
b) How much of the final amount is from deposits and how much from interest?

a) $A = P \cdot \dfrac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}}$

$4000 = P \dfrac{\left(1 + \frac{0.0275}{52}\right)^{52 \cdot 2} - 1}{\frac{0.0275}{52}}$

$4000 = P \cdot 106.88\ldots$

$P = \dfrac{4000}{106.88\ldots} = 37.42$

b) Total deposits $= 37.42 \cdot 52 \cdot 2$
$= 3891.68$

From interest $= 4000 - 3891.68$
$= 108.32$

7. (32pts) Interest rates for home mortgages are at their historic lows, making it a good time to buy a home. Suppose your friend takes out a 30-year loan for \$180.000 at 3.5% compounded monthly.

a) What is her montly payment on the loan?

b) What are her total payments over the course of the loan? How much of this amount is for interest?

c) How much of her first payment goes toward interest, and how much towards the principal?

d) How much does she owe after 18 years?

a) $P = PMT \cdot \dfrac{1-\left(1+\frac{r}{4}\right)^{-4t}}{\frac{r}{4}}$

$180{,}000 = PMT \cdot \dfrac{1-\left(1+\frac{0.035}{12}\right)^{-12\cdot 30}}{\frac{0.035}{12}}$

$180{,}000 = PMT \cdot 222.69\ldots$

$PMT = \dfrac{180{,}000}{222.69\ldots} = 808.28$

b) total payments $= 808.28 \cdot 12 \cdot 30$

$= 290{,}980.80$

toward interest $= 290{,}980.80 - 180{,}000$

$= 110{,}980.80$

c) $180{,}000 \cdot 0.035 \cdot \frac{1}{12} = 525$ interest on 1st payment

$808.28 - 525 = 283.28$ toward principal

d) $P = PMT \dfrac{1-\left(1+\frac{r}{4}\right)^{-4t}}{\frac{r}{4}}$ ← time remaining

$P = 808.28 \cdot \dfrac{1-\left(1+\frac{0.035}{12}\right)^{-12\cdot 12}}{\frac{0.035}{12}}$

$= 808.28 \cdot 117.44\ldots$

$= 94{,}929.41$

Amount owed after 18 years (still more than half the loan)

8. (12pts) If you save for retirement by depositing \$300 every month into an account bearing 8.22% interest, compounded monthly, how long will it take until you have \$200,000 in the account?

$$A = P\,\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$$

$$200000 = 300\cdot\frac{\left(1+\frac{0.0822}{12}\right)^{12\cdot t}-1}{\frac{0.0822}{12}} \quad \mid \div 300$$

$$666.66\ldots = \frac{(1.00685)^{12t}-1}{0.00685} \quad \mid \cdot 0.00685$$

$$4.566\ldots = (1.00685)^{12t}-1 \quad \mid +1$$

$$5.566\ldots = (1.00685)^{12t} \quad \mid \log$$

$$\log(5.566) = \log(1.00685)^{12t}$$

$$\log(5.566) = 12t\log(1.00685)$$

$$t = \frac{\log(5.566)}{12\log(1.00685)}$$

$$= 20.957053 \text{ years}$$

about 21 years.

Bonus. (10pts) A 20-year \$100,000 mortgage has a monthly payment of \$567.20 at interest rate 3.25%, compounded monthly (you don't need to verify this). Banks allow you to pay more than the monthly rate in an effort to pay off the loan early. If the borrower makes a monthly payment of \$700, how long will it take them to pay off this loan? *(Hint: use only the loan formula.)*

$$A = P\,\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$$

$$100{,}000 = 700\cdot\frac{1-\left(1+\frac{0.0325}{12}\right)^{-12\cdot t}}{\frac{0.0325}{12}} \quad \mid \div 567.20$$

$$142.85\ldots = \frac{1-(1.0027\ldots)^{-12t}}{0.0027\ldots} \quad \mid \cdot 0.0027\ldots$$

$$0.386\ldots = 1-(1.0027\ldots)^{-12t} \quad \mid -1$$

$$-0.613\ldots = -(1.0027\ldots)^{-12t} \quad \mid \cdot(-1)$$

$$0.613 = (1.0027\ldots)^{-12t} \quad \mid \log$$

$$\log 0.613\ldots = \log(1.0027)^{-12t}$$

$$\log 0.613\ldots = -12t\log(1.0027\ldots)$$

$$t = \frac{\log 0.613\ldots}{-12\log(1.0027\ldots)}$$

$$= 15.073760 \text{ years}$$

About 15 years