

Find the following antiderivatives.

1. (4pts) $\int 4x^2 - 3\sqrt{x} dx =$

2. (4pts) $\int e^{3x+1} dx =$

3. (4pts) $\int \sec(4x) \tan(4x) dx =$

4. (15pts) Find $\int_{-2}^2 x - 1 dx$ in two ways (they'd better give you the same answer!):
- Using the “area” interpretation of the integral. Draw a picture.
 - Using the Fundamental Theorem of Calculus.

5. (6pts) Write in sigma notation.

$$\frac{2}{3} + \frac{4}{9} + \frac{6}{27} + \frac{8}{81} + \frac{10}{243} =$$

6. (10pts) Evaluate the definite integral:

$$\int_1^4 \frac{x^2 - 1}{\sqrt{x}} dx =$$

Use the substitution rule in the following integrals:

7. (9pts) $\int (6x^2 - 4x)(x^3 - x^2 + 2)^5 dx =$

8. (10pts) $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sec^2 x}{\tan^7 x} dx =$

9. (8pts) Find the function f if $f'(x) = \frac{5}{x^4} - \frac{3}{x}$, and $f(1) = 7$.

10. (22pts) The function $f(x) = \cos x$, $0 \leq x \leq \frac{\pi}{2}$ is given.

a) Write down the expression that is used to compute R_4 . Then compute R_4 .

b) Illustrate with a diagram, where appropriate rectangles are clearly visible. What does R_4 represent?

c) Is R_4 an overestimate or an underestimate of the area?

d) Using the Fundamental Theorem of Calculus, evaluate $\int_0^{\frac{\pi}{2}} \cos x \, dx$. What is the error of R_4 ?

11. (8pts) Show that $0.82 \leq \int_1^{1.3} e^{x^2} dx \leq 1.63$. (Note: the antiderivative of e^{x^2} cannot be found among elementary functions, so don't try to do it by evaluating the integral.)

Bonus. (10pts) The gist of section 5.5 is this:

\int_a^b rate of change of $F =$ change of F from a to b . In other words, $\int_a^b F'(x) dx = F(b) - F(a)$.

Use the fact above to solve the following problem. Water flows in and out of a tank at rate $3 - \frac{1}{2}t$ liters/minute. There were 5 liters of water in the tank at time $t = 0$.

- a) By how much does the amount of water in tank change from $t = 0$ to $t = 4$?
- b) How much water is in the tank at time $t = 4$?
- c) By how much does the amount of water in tank change from $t = 4$ to $t = 10$?
- d) How much water is in the tank at time $t = 10$?