

Find the limits. You may use L'Hopital's rule.

1. (10pts) $\lim_{x \rightarrow \infty} \frac{x^2 + 3x - 7}{e^x} =$

Which grows faster, $x^2 + 3x - 7$ or e^x ?

2. (8pts) $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2} =$

3. (10pts) $\lim_{x \rightarrow 0^+} \sqrt[5]{x} \ln x =$

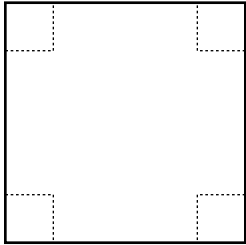
4. (10pts) $\lim_{x \rightarrow \infty} x^{\frac{1}{x}} =$

5. (32pts) Let $f(x) = \ln(x^2 + 9)$. Draw an accurate graph of f by following the guidelines.
- Find the intervals of increase and decrease, and local extremes.
 - Find the intervals of concavity and points of inflection.
 - Find $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$.
 - Use information from a)–d) to sketch the graph

6. (8pts) Find the limit without using L'Hopital's rule.

$$\lim_{x \rightarrow \infty} \frac{5x^2 - 3x + 1}{2x^2 - x + 2} =$$

7. (22pts) A square piece of cardboard has side length 4ft. Four equal-size square pieces are cut from the corners to produce a cross-like shape, whose "flaps" are lifted up to form a box without a top. Find the size of the cutout that produces the box with the largest volume.



Bonus. (10pts) Draw a line of negative slope through the point $(2, 3)$. Along with the axes, this line forms a right triangle in the first quadrant. Among all triangles obtained in this way, find the one with the smallest area.