Spring '10/MAT 250/Exam 1 Name: *Show all your work.*

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x\to 1^+} f(x) =$

$\lim_{x\to 3^-} f(x) =$

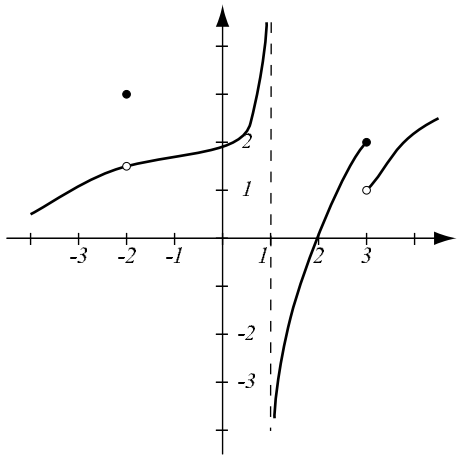
$\lim_{x\to 3^+} f(x) =$

$\lim_{x\to 3} f(x) =$

$\lim_{x\to -2} f(x) =$

$f(-2) =$

List points where f is not continuous and justify why it is not continuous at those points.



2. (4pts) Find the following limit algebraically (no need to justify, other than showing the computation).

$\lim_{x\to 2} (x^2 + 4x - 6) =$

3. (8pts) Let $\lim_{x\to 3} f(x) = 4$ and $\lim_{x\to 3} g(x) = 7$. Use limit laws to find the limit below and show each step.

$$\lim_{x\to 3} \frac{3f(x) + 7}{x^2 - f(x)g(x)} =$$

4. (10pts) Find the domain of $f(x) = \dfrac{x^2}{\sqrt{e^x}}$. Then explain, using continuity laws, why the function is continuous on its domain.

5. (16pts) The height of a turnip t seconds after getting thrown upwards with initial velocity 20 meters per second is given by $h(t) = 20t - 5t^2$ (in meters).
a) Find the average velocities of the object over six short intervals of time, three of them beginning with 1.5, and three ending with 1.5. Show the table of values.
b) Use the information in a) to find the instantaneous velocity of the turnip at $t = 1.5$.

6. (10pts) Find the following limit algebraically (do not use the calculator) and justify.

$$\lim_{x \to 4-} \frac{5x + 3}{x - 4} =$$

7. (18pts) The equation $e^x = x^2$ is given.
a) Use the Intermediate Value Theorem to show that this equation has at least one solution. Write a nice sentence that shows how you are using the IVT.
b) Use the bisection method and your calculator to find an interval of width less than 0.05 that contains your solution. Show every intermediate step.

8. (8pts) Let $f(x) = x^2 - 3x + 5$, and let $P = (4, 9)$. If $Q = (x, f(x))$ is another (general) point on the graph of f, write the formula for the slope of the secant line PQ and simplify.

9. (10pts) Consider the limit $\lim_{x\to 0} \dfrac{\sin x - x}{x^3}$.
a) Use your calculator to find the limit correct to six decimal places — write down the table on paper. Make a guess as to what the limit is exactly.
b) What does the calculator give you if you take an x very close to 0? Does this alter your estimate of the limit? Why or why not?

Bonus. (10pts) Below is the graph of the outdoor temperature (in °C) measured t hours after noon. Use it to put the following three numbers in increasing order. No computation is needed, but justify your answer.
a = average rate of change from $t = 1$ to $t = 2$
b = average rate of change from $t = 1$ to $t = 3$
c = instantaneous rate of change at $t = 1$