Calculus 1 — Exam 1
MAT 250, Spring 2011 — D. Ivanšić

Name: Solution

Show all your work!

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x\to -2^-} f(x) = 3$

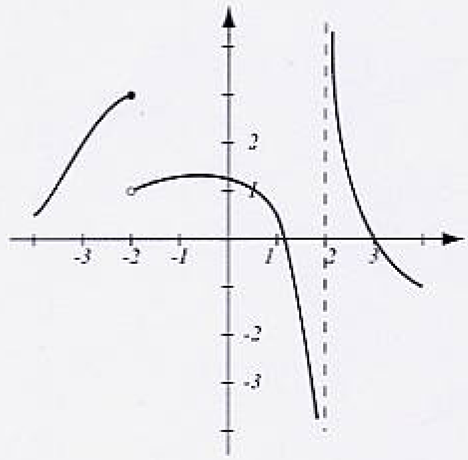
$\lim_{x\to -2^+} f(x) = 1$

$\lim_{x\to -2} f(x) =$ d.n.e one-sided limits not equal

$\lim_{x\to 2^-} f(x) = -\infty$

$\lim_{x\to 2^+} f(x) = \infty$

$\lim_{x\to 2} f(x) =$ d.n.e. one sided limits not equal



List points where f is not continuous and justify why it is not continuous at those points.

Not continuous at

$x=-2$ because $\lim_{x\to -2} f(x)$ d.n.e

$x=2$ because $\lim_{x\to 2} f(x)$ d.n.e.

2. (4pts) Find the following limit algebraically (no need to justify, other than showing the computation).

$\lim_{x\to 2} (x^2+4x-6) = 2^2+4\cdot 2-6 = 6$

3. (8pts) Let $\lim_{x\to 4} f(x) = 1$ and $\lim_{x\to 4} g(x) = -2$. Use limit laws to find the limit below and show each step.

$$\lim_{x\to 4} \frac{x^2-2g(x)}{2f(x)-g(x)+7} = \frac{\lim_{x\to 4}(x^2-2g(x))}{\lim_{x\to 4}(2f(x)-g(x)+7)} = \frac{\lim_{x\to 4} x^2 - \lim_{x\to 4}(2g(x))}{\lim_{x\to 4}(2f(x)) - \lim_{x\to 4} g(x) + \lim_{x\to 4} 7}$$

$$= \frac{\left(\lim_{x\to 4} x\right)^2 - 2\lim_{x\to 4} g(x)}{2\lim_{x\to 4} f(x) - (-2)+7} = \frac{4^2-2\cdot(-2)}{2\cdot 1+2+7} = \frac{20}{11}$$

4. (12pts) Let $f(x) = \dfrac{\sin x}{x}$.
a) Find the domain of f.
b) Explain, using continuity laws, why the function is continuous on its domain.
c) At points of discontinuity, state the type of discontinuity (jump, infinite, removable) and explain. Use a well-known limit that we mentioned for this.

a) $x \neq 0$
Domain $= (-\infty, 0) \cup (0, \infty)$

b) $\sin x$ is continuous on $\mathbb{R}$
x is continuous

$\frac{\sin x}{x}$ is continuous as a quotient of two continuous functions, as long as $x \neq 0$

c) $x = 0$ only possible point of discontinuity.

$\lim_{x \to 0} \frac{\sin x}{x} = 1$ (saw in class)

If we set $f(0) = 1$, we'll have a continuous function (so: removable discontinuity)

5. (16pts) The height of a blackberry (fruit or phone — your choice!) t seconds after getting thrown upwards with initial velocity 30 meters per second is given by $h(t) = 30t - 5t^2$ (in meters).
a) Find the average velocities of the blackberry over six short intervals of time, three of them beginning with 2, and three ending with 2. Show the table of values. What are the units?
b) Use the information in a) to find the instantaneous velocity of the blackberry at $t = 2$. What are the units?

$h(2) = 40$

a) $\dfrac{\Delta h}{\Delta t} = \dfrac{h(t) - h(2)}{t-2} = \dfrac{30t - 5t^2 - 40}{t-2}$

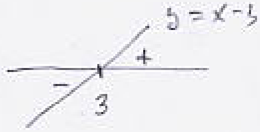
t	average velocity on $[2,t]$ or $[t,2]$
1.9	10.5
1.99	10.05
1.999	10.005
2.1	9.5
2.01	9.95
2.001	9.995

in meters/second

It appears instantaneous velocity is 10 m/s

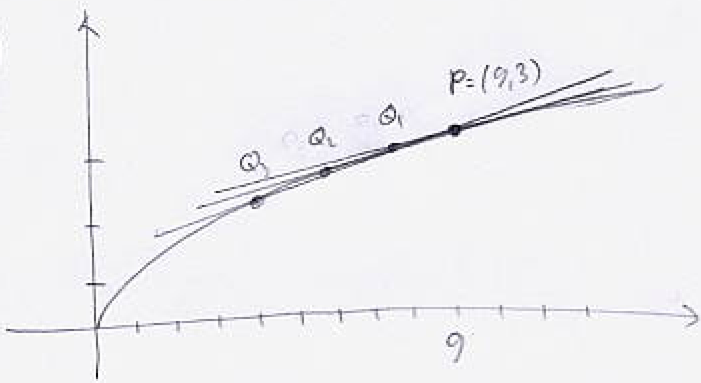
6. (10pts) Find the following limit algebraically (do not use the calculator) and justify.

$$\lim_{x\to 3+} \frac{2x-7}{x-3} = \frac{2\cdot 3-7}{0+} = \frac{-1}{0+} = -\infty$$



7. (18pts) Let $f(x) = \sqrt{x}$, and let $P = (9, 3)$.
a) Draw the graph of f on the interval $[0, 11]$.
b) Draw three secant lines PQ, where Q is to the left of P.
c) If $Q = (x, f(x))$ is a general point on the graph of f, write the formula for the slope of the secant line PQ.
d) Find slopes of three secant lines where Q is close to P (show table) and use those slopes to find the slope of the tangent line at P.

a)
b)



c) $m = \frac{f(x)-3}{x-9} = \frac{\sqrt{x}-3}{x-9}$

d)

x	$m_{PQ} = \frac{\sqrt{x}-3}{x-9}$
8.9	0.1671322
8.99	0.166713
8.999	0.166671

Slope of tangent line is approx. 0.16667...
(possibly $\frac{1}{6}$)

8. (16pts) Consider the limit $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$.

a) Can you use a limit law to find the limit? Why or why not?

b) Use your calculator to estimate the limit — write down the table on paper. Make a guess as to what the limit is exactly.

b) What does the calculator give you if you take an x very close to 0? Does this alter your estimate of the limit? Why or why not?

a) We cannot use the quotient limit law since the limit in the denominator is 0, so the limit law cannot be applied.

b)

x	$\frac{e^x-1-x}{x^2}$
0.01	0.501671
0.001	0.500167
0.0001	0.500020
10^{-5}	0.5
10^{-6}	0.5
10^{-7}	0
10^{-8}	0

It appears limit is $0.5 = \frac{1}{2}$

c) When x is very small, calculator's approximation of e^x is equal to $1+x$, so e^x-1-x gives 0.

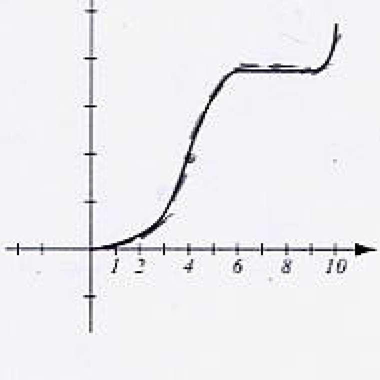
The limit is more likely to be $\frac{1}{2}$, since our experience tells us that the calculator sometimes makes rounding errors, causing unexpected values in the table.

Bonus. (10pts) Below is the graph of the position of a car t minutes after noon. Answer the following, with justification.

a) Is there a time interval when the car is not moving? If so, when?

b) When is the car speeding up? Give time intervals.

c) When is the car slowing down? Give time intervals.



Velocity = slope of tangent line to graph of position

a) Car not moving $\Leftrightarrow$ velocity $= 0$

Happens on interval $[6, 9]$

b) Slopes of tangent lines are increasing on $[0, 4]$ and $[9, 10]$ $\leftarrow$ when car is speeding up

c) Slopes of tangent lines are decreasing on $[4, 6]$ $\leftarrow$ when car is slowing down.