Mathematical Concepts — Joysheet 10
MAT 117, Spring 2011 — D. Ivanšić

Name: Solution

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1. (26pts) An automobile insurance company is deciding which animated character to use as its spokescritter. The preference rankings of the board members are shown below, from the choices: cocker-spaniel, firefly and ostrich.

Votes:	7	2	5	3	4	3
1st	C	C	F	F	O	O
2nd	F	O	C	O	C	F
3rd	O	F	O	C	F	C

a) Which choice wins the vote in a plurality election?
b) Which choice wins the vote in a plurality election with a runoff?
c) Which choice is the pairwise comparison winner?
d) Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

a) C: 7+2 = 9 wins
F: 5+3 = 8
O: 4+3 = 7

b) C and F in runoff:
C: 9+4 = 13 wins
F: 8+3 = 11

c) C: 7+2+4 = 13 wins
F: 5+3+3 = 11

C: 7+2+5 = 14 wins
O: 4+3+3 = 10

F: 5+3+7 = 15 wins
O: 4+3+2 = 9

Pairwise tally: C: 2 wins
F: 1 win ⇒ C wins pairwise comparison
O: 0 wins

d) C: 9·3 + 9·2 + 6·1 = 51 wins
F: 8·3 + 10·2 + 6·1 = 50
O: 7·3 + 5·2 + 12·1 = 43
144

7+2+5+3+4+3 = 24 voters ↘ equal
each contributes 6 pts; 24·6 = 144

2. (34pts) A poll of members of a community asked people to rank their preferences on a curfew for young people under the age of 18. The choices were 10PM, 12AM, 1AM, and no curfew. Their preference rankings broke down into the following percentages.

Votes	9	2	6	20	4	17	10	32
1st	10P	12A	12A	12A	1A	1A	1A	no
2nd	12A	10P	1A	1A	12A	12A	no	1A
3rd	1A	1A	10P	no	10P	no	12A	12A
4th	no	no	no	10P	no	10P	10P	10P

a) Which choice wins the vote in a plurality election?
b) Which choice wins the vote in a plurality election with elimination (see book)?
c) Which choice is the pairwise comparison winner?
d) Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

a) 10P : 9 = 9
12A : 2+6+20 = 28
1A : 4+17+10 = 31
no : 32 = 32 wins

b) In first round 10P is eliminated.

12A : 28 + 9 = 37
1A : 31 = 31 ← eliminated
no : 32 = 32

Third round:

12A : 37 + 4 + 17 = 58 wins
no : 32 + 10 = 42

c) {10P : 9 = 9
12A : 28+4+17+32 = 91 wins

{12A : 28+9 = 37
1A : 31+32 = 63 wins

{10P : 9+2 = 11
1A : 31+6+20+32 = 89 wins

{12A : 28+9+4+17 = 58 wins
no : 32+10 = 42

{10P : 9+2+6+4 = 21
no : 32+20+17+10 = 79 wins

{1A : 31+9+2+6+20 = 68 wins
no : 32+ = 32

Pairwise comparison
Point tally: 10P : 0
12A : 2
1A : 3 → 1A wins
no : 1

d) 10P : 9·4 + 2·3 + 10·2 + 79·1 = 141
12A : 28·4 + 30·3 + 42·2 + 0·1 = 286
1A : 31·4 + 58·3 + 11·2 + 0·1 = 320 wins
no : 32·4 + 10·3 + 37·2 + 21·1 = 253
1000

100 voters, each gives 4+3+2+1 pts = 10 pts
100 · 10 = 1000