Mathematical Concepts — Joysheet 7
MAT 117, Spring 2011 — D. Ivanšić

Name: Solution

Show all your work!

1. (18pts) A bag contains four red balls and two green balls. Two balls are drawn without replacement. (For this problem distinguish between the balls of the same color.)

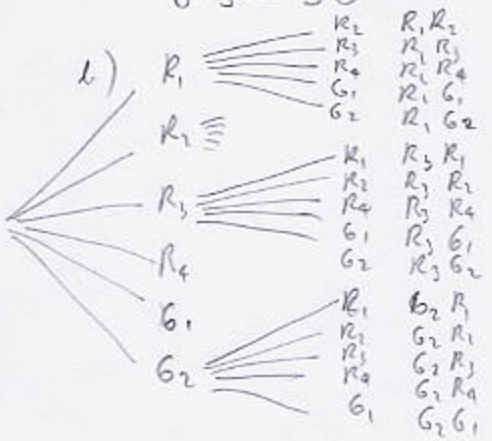
a) Determine the number of points in the sample space.
b) Construct a tree diagram (you can draw it only partially) and list the sample space.

Determine the probability that:
c) the first ball was red and the second was green.
d) exactly one of the balls was red.
e) exactly one of the balls was green.
f) at least one ball was green.

a) First ball chosen in 6 ways,
Second ball chosen in 5 ways

$6 \cdot 5 = 30$

b)



c) $P(\text{1st red and 2nd green}) = \frac{4 \cdot 2}{30} = \frac{8}{30} = \frac{4}{15} = 0.266667$

d) $P(\text{exactly one red}) = \frac{4 \cdot 2 + 2 \cdot 4}{30} = \frac{16}{30} = \frac{8}{15} = 0.533333$

e) $P(\text{exactly one green}) = P(\text{exactly one is red}) = \frac{8}{15} = 0.533333$

since: if exactly one is red, then exactly one is green

f) $P(\text{at least one green}) = 1 - P(\text{none green})$
$= 1 - \frac{4 \cdot 3}{30} = \frac{30}{30} - \frac{12}{30} = \frac{18}{30} = \frac{3}{5} = 0.6$

2. (13pts) At a popular restaurant, the probability of getting seating on a Saturday within a half-hour of arrival is 35%. Assume that getting seating on different Saturdays are independent events. What is the probability of
a) getting seating on two different Saturdays?
b) getting seating at least once on two different Saturdays?
c) not getting seating on five different Saturdays?

a) $P(\text{seat on 1st Sat and 2nd Sat.}) = P(\text{1st Sat}) \cdot P(\text{2nd Sat})$
$= 0.35 \cdot 0.35 = 0.1225$

b) $P(\text{at least once on two Sat's}) = 1 - P(\text{not once on two Sat}) = 1 - 0.4225 = 0.5775$

$P(\text{not once on two Sat}) = P(\text{not 1st Sat and not 2nd Sat}) =$
$= P(\text{not 1st Sat}) \cdot P(\text{not 2nd Sat}) = 0.65 \cdot 0.65 = 0.4225$

c) $P(\text{not on five Sat's}) = P(\text{not on 1st AND not on 2nd AND ... AND not on 5th}$
$= P(\text{not on 1st}) \cdot P(\text{not on 2nd}) \cdot \ldots \cdot P(\text{not on 5th})$
$= 0.65^5 = 0.116029$

3. (5pts) A women's shoe store has 45 boxes of shoes that are flats or have a peep-toe. If 32 boxes have shoes with a peep-toe and 24 boxes have flats, what is the probability that a randomly chosen box contains flats with a peep-toe?

$P(\text{peep-toe or flats}) = P(\text{peep-toe}) + P(\text{flats}) - P(\text{peep-toe and flats})$

$\frac{45}{45} = \frac{32}{45} + \frac{24}{45} - P(\text{peep-toe and flats})$

$\frac{45}{45} = \frac{56}{45} - P \quad \left| -\frac{56}{45} \right. \quad -\frac{11}{45} = -P \quad P(\text{peep-toe and flats}) = \frac{11}{45}$

4. (12pts) Two cards are drawn from a standard deck, without replacement. What is the probability that:

a) both cards are face cards?

b) neither card is a number between 4 and 7?

c) at least one card is an ace?

not independent events

a) $P(\text{1st face and 2nd face}) = P(\text{1st face}) \cdot P(\text{2nd face} \mid \text{1st face}) = \frac{12}{52} \cdot \frac{11}{51} = \frac{3}{13} \cdot \frac{11}{17} = \frac{11}{221} = 0.049774$

4-7 ← 16 cards

not in 4-7 ← 52-16=36 cards

b) $P(\text{1st not in 4-7 AND 2nd not in 4-7})$

$= P(\text{1st not in 4-7}) \cdot P(\text{2nd not in 4-7} \mid \text{1st not in 4-7})$

$= \frac{36}{52} \cdot \frac{35}{51} = \frac{9}{13} \cdot \frac{5}{17} = \frac{45}{221} = 0.203620$

c) $P(\text{at least one is ace}) = 1 - P(\text{neither is ace}) = 1 - \frac{188}{221} = \frac{33}{221} = 0.149321$

$= P(\text{1st not ace and 2nd not ace}) = P(\text{1st not ace}) \cdot P(\text{2nd not ace} \mid \text{1st not ace})$

$= \frac{48}{52} \cdot \frac{47}{51} = \frac{4}{13} \cdot \frac{47}{17} = \frac{4 \cdot 47}{13 \cdot 17}$

5. (12pts) The table below shows the pattern of payment methods at a certain grocery store on one fine Wednesday. What is the probability that a random shopper:

a) paid by cash or check?

b) was over 40?

c) paid by credit card, given they were under 40?

d) paid by debit card, given they were over 40?

Age	Cash or Check	Debit Card	Credit Card	Total
under 40	15	32	51	98
over 40	35	21	27	83
Total	50	53	78	181

a) $\frac{50}{181} = 0.276243$

b) $\frac{83}{181} = 0.458564$

c) $\frac{51}{98} = 0.520408$

d) $\frac{21}{83} = 0.253012$