

1. (8pts) A curious observer counts the number and type of vehicles passing through a street on several Wednesday afternoons. She obtains the following results:

Vehicle	car	SUV	pick-up	heavy truck
Number	67	53	59	33

total:  
212

If a random vehicle is observed on a Wednesday afternoon, what is the empirical probability that it is

- a) an SUV?  
b) a heavy truck?  
c) not a car?

$$a) \frac{53}{212} = 0.25$$

$$b) \frac{33}{212} = 0.155660$$

$$c) P(\text{not car}) = 1 - P(\text{car}) = 1 - \frac{67}{212} = \frac{212}{212} - \frac{67}{212} = \frac{145}{212} = 0.683962$$

2. (17pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

Event	probability	odds against	odds in favor
a) Rolling a 3 on a die	$\frac{1}{6}$	5 : 1	1 : 5
b) Drawing a 7 or a 9 from a deck of cards	$\frac{8}{52} = \frac{2}{13}$	11 : 2	2 : 11
c) Drawing a red face card from a deck of cards	$\frac{6}{52} = \frac{3}{26}$	23 : 3	3 : 23
d) Getting two heads on two coin tosses	$\frac{1}{4}$	3 : 1	1 : 3
e) Getting sum 3 or 10 on a roll of two dice	$\frac{5}{36}$	31 : 5	5 : 31

b) There are 4 sevens and 4 nines in a deck

c) There are  $2 \cdot 3 = 6$  red face cards

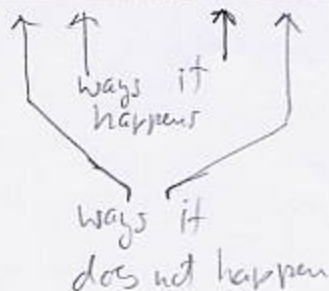
d) HH  
HT  
TH  
TT

$$e) 3 = 2+1$$

$$10 = 4+6$$

$$6+5$$

$$5+5$$



3. (4pts) The odds against rain tomorrow are 7-to-2.
- a) What is the probability that it rains tomorrow?
- b) What is the probability that it does not rain tomorrow?

not happen  
↓  
happen

$$a) P(\text{rain}) = \frac{2}{9}$$

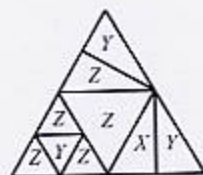
$$b) P(\text{no rain}) = \frac{7}{9}$$

4. (4pts) The probability of seeing a rainbow on May 1st is  $\frac{2}{99}$ .
- a) What are the odds against seeing a rainbow?
- b) What are the odds in favor of seeing a rainbow?

$$a) 97:2$$

$$b) 2:97$$

5. (13pts) A bean bag is tossed onto the triangular field shown. What is the probability that the bean bag lands on:



- a) a field labeled X?
- b) on a field labeled Y?
- c) a field labeled X or Z?
- d) What are the odds against landing on X or Y?

$$d) P(X \text{ or } Y) = \frac{1}{8} + \frac{5}{16} = \frac{2+5}{16} = \frac{7}{16}$$

$$\left. \begin{array}{l} \text{odds against } (X \text{ or } Y) \\ = 9:7 \end{array} \right\}$$

$P(S) = \text{ratio of area of } S \text{ to area of entire triangle}$

$$a) P(X) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8} = 0.125$$

$$b) P(Y) = \frac{1}{4} \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{8} + \frac{1}{8} + \frac{1}{16} = \frac{2+2+1}{16} = \frac{5}{16} = 0.3125$$

$$c) P(X \text{ or } Z) = \frac{1}{4} \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{3}{4} + \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8} + \frac{1}{16} + \frac{3}{16} + \frac{1}{8} = \frac{2+4+3+2}{16} = \frac{11}{16} = 0.6875$$

6. (14pts) At a charity event, the following game of chance involving a single die is played: a player pays \$5 and collects \$15 if they roll a 6, \$7 if they roll a 2 or a 4, and nothing if they roll a 1, 3 or a 5.

- a) Find the expected value of this game.
- b) What is the fair price of this game?
- c) The event organizers expect that 150 plays will be made. How much do they expect to collect from this game? How much should they charge per play in order to expect collecting \$100 from this game?

$$a) E = P(\text{roll } 6) \cdot (15-5) + P(\text{roll } 2 \text{ or } 4) \cdot (7-5) + P(\text{roll } 1, 3 \text{ or } 5) \cdot (-5)$$

$$= \frac{1}{6} \cdot 10 + \frac{2}{6} \cdot 2 + \frac{3}{6} \cdot (-5) = \frac{10+4-15}{6} = -\frac{1}{6} = -0.166667$$

Player expects to lose about 17c per game.

$$b) \text{Fair price} = -\frac{1}{6} + 5 = -\frac{1}{6} + \frac{10}{6} = \frac{29}{6} = 4.833333$$

$$c) \text{Per game, organizer expects to gain } \frac{1}{6}$$

For 150 games this is  $150 \cdot \frac{1}{6} = \$25$ .

If they would like to collect \$100, they need an extra \$75, to be gained over 150 games, or  $\frac{75}{150} = 0.50$  per game. Thus, they should charge \$5.50.