Mathematical Concepts — Joysheet 3
MAT 117, Spring 2011 — D. Ivanšić

Name: Solution

Show all your work!

1. (12pts) To save for a new car, Alison deposits \$400 every month into an account bearing 4.25% interest, compounded monthly.
a) How much does she have in the account in 5 years?
b) How much did she earn in interest over these 5 years?

a) $A = P\dfrac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$

$A = 400\dfrac{\left(1+\frac{0.0425}{12}\right)^{12\cdot 5}-1}{\frac{0.0425}{12}}$

$A = 400 \cdot 66.7205\ldots$

$A = 26{,}688.21$

b) Total deposits: $60 \cdot 400 = 24000$

Interest earned $= 26{,}688.21 - 24{,}000$

$= 2{,}688.21$

2. (10pts) In order to buy a \$4,000 computer in two years, Joe decides to save up money. If he can deposit money into an account bearing 3%, compounded quarterly, how much should he deposit into the account every quarter?

$A = P\dfrac{\left(1+\frac{r}{n}\right)^{4t}-1}{\frac{r}{n}}$

$4000 = P \cdot \dfrac{\left(1+\frac{0.03}{4}\right)^{4\cdot 2}-1}{\frac{0.03}{4}}$

$4000 = P \cdot 8.21\ldots \quad | \div 8.21\ldots$

$P = \dfrac{4000}{8.21\ldots} = 487.02$

3. (12pts) The Patels would like to save \$50,000 for a down payment on a house. If they can set aside \$500 every month into an account bearing 4.75%, compounded monthly, how long will it take them to save the desired amount?

$A = P\dfrac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$

$50000 = 500\dfrac{\left(1+\frac{0.0475}{12}\right)^{12t}-1}{\frac{0.0475}{12}} \quad | \div 500$

$100 = \dfrac{(1.0039\_)^{12t}-1}{0.0039\_} \quad | \cdot 0.0039\_$

$0.3958\_ = (1.0039\_)^{12t}-1 \quad | +1$

$1.3958\_ = (1.0039\_)^{12t} \quad | \log(\ )$

$\log 1.3958\_ = \log(1.0039\_)^{12t}$

$\log 1.3958\_ = 12t \log 1.0039\_ \quad | \div 12\log(1.0039\_)$

$t = \dfrac{\log 1.3958\_}{12 \log 1.0039\_} = 7.034762$

≈ 7.03 years

4. (16pts) At the time of little Ella's birth, her parents had \$5,000 in savings. They would like to have \$100,000 for Ella's college by the time she is 18. They can get 5.5% interest in an account compounded quarterly, and they invest the \$5,000 in this account. How much should they additionally deposit into this account every quarter in order to have the target amount? Including the \$5,000, what was the total amount of their deposits? *Hint: treat this situation as two different accounts, whose goal is to have a total of \$100,000 between them.*

Value of 5000 in 18 years:

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = 5000\left(1+\frac{0.055}{4}\right)^{4\cdot 18}$

$= 5000 \cdot 2.673\ldots$

$= 13,365.72$

$100,000 - 13,365.72$

$= 86,634.28$

Adjusted target

$A = P\dfrac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$

$86,634.28 = P \cdot \dfrac{\left(1+\frac{0.055}{4}\right)^{4\cdot 18}-1}{\frac{0.055}{4}}$

$86,634.28 = P \cdot 121.68\ldots \quad \left| \div 121.68\ldots \right.$

$P = \dfrac{86,634.28}{121.68} = 711.9656\ldots$

$P = \$\,711.97$

Total deposits: $5000 + 711.97 \cdot 72 = 56,261.84$

5. (10pts) Little Jenna, who was in the same maternity ward as Ella, was born to poor parents who were not able to save for her college. After high school, determined to go to college, Jenna takes out a student loan for \$85,000, with an interest rate of 5.5%, compounded quarterly. She repays this loan over 18 years with equal quarterly payments. What is the amount she pays every quarter?

$P = m\dfrac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$

$85,000 = m\dfrac{1-\left(1+\frac{0.055}{4}\right)^{-4\cdot 18}}{\frac{0.055}{4}}$

$85000 = m \cdot 45.52\ldots \quad \left| \div 45.52 \right.$

$m = \dfrac{85,000}{45.52\ldots} = 1867.28$