Mathematical Concepts — Final Exam **MAT 117, Spring 2011 — D. Ivanšić**	**Name:** Solution *Show all your work!*

$i = prt$ $A = p(1+rt)$ $A = p\left(1+\frac{r}{n}\right)^{nt}$ $A = p\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$ $P = m\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$ $APY = \left(1+\frac{r}{n}\right)^{n}-1$

$\frac{a}{b} = \frac{1-P(E)}{P(E)}$ $P(E) = \frac{b}{a+b}$ where odds against E are $a:b$ $P(B|A) = \frac{n(A \text{ and } B)}{n(A)}$

$P(A \text{ or } B) = P(A)+P(B)-P(A \text{ and } B) = P(A)+P(B)$ (if A and B are mutually exclusive)

$P(A \text{ and } B) = P(A)\cdot P(B|A)$ $P(A \text{ and } B) = P(A)\cdot P(B)$ if A and B are independent

$E = P_1\cdot A_1 + P_2\cdot A_2 + \cdots + P_n\cdot A_n$

angle = (relative frequency) $\cdot$ 360° midrange = $\frac{\text{lowest value} + \text{highest value}}{2}$

$\overline{x} = \frac{x_1+x_2+\cdots+x_n}{n} = \frac{\sum_i x_i}{n}$ range = highest value − lowest value

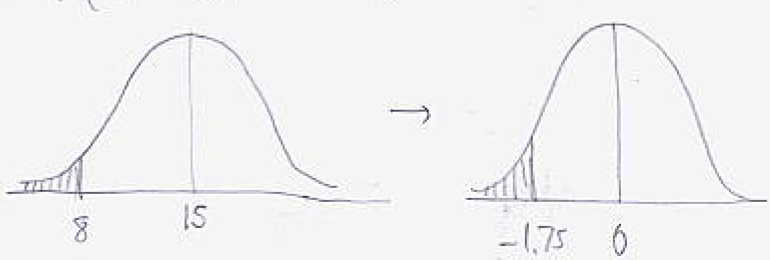
$s = \sqrt{\frac{(x_1-\overline{x})^2+(x_2-\overline{x})^2+\cdots+(x_n-\overline{x})^2}{n-1}} = \sqrt{\frac{\sum_i (x_i-\overline{x})^2}{n-1}}$ $Z = \frac{X-\mu}{\sigma}$

1. (13pts) Assume the number of hours college students spend working per week is normally distributed with a mean of 15 hours and standard deviation 4 hours. Draw pictures showing which area you are computing as you answer:

a) What percentage of students work fewer than 8 hours per week?

b) What percentage of students work between 13 and 17 hours per week?

a) $P(X \leq 8) = A(Z \leq -1.75) = 0.0401$ $\qquad Z = \frac{8-15}{4} = -1.75$



4.01% work fewer than 8 hours per week.

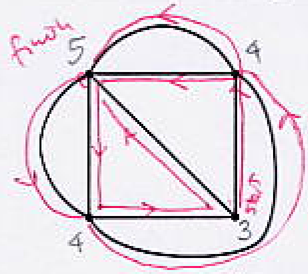
b) $P(13 \leq X \leq 17) = A(-0.5 \leq X \leq 0.5)$ $\qquad Z = \frac{13-15}{4} = -0.5$

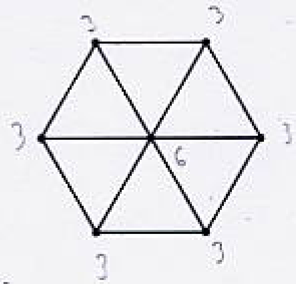
$= A_2 - A_1 = 0.6915 - 0.3085$ $\qquad Z = \frac{17-15}{4} = 0.5$

$= 0.3830$

38.30% work between 13 and 17 hours per week.

2. (12pts) Determine whether each of the following graphs has an Euler path or an Euler circuit. If it does, find it, if not, explain why not.





Has more than two odd vertices

⇒ has neither Euler path nor circuit

Has two odd vertices

⇒ It has an Euler path

but not an Euler circuit

3. (23pts) Fans of the "Star Wars" saga were asked to elect their favorite episode of the original series (1977-1983). The rankings of the group are below.

Votes:	12	4	11	7	6	3
1st	IV	IV	V	V	VI	VI
2nd	V	VI	IV	VI	IV	V
3rd	VI	V	VI	IV	V	IV

→ 43 voters

a) Which choice wins the vote in a plurality election?
b) Which choice wins the vote in a plurality election with a runoff?
c) Which choice is the pairwise comparison winner?
d) Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

a) IV: $12+4=16$
V: $11+7=18$ wins
VI: $6+3=9$

c) IV: $16+6=22$ wins IV: $16+11=27$ wins V: $18+12=30$
V: $18+3=21$ VI: $9+7=16$ VI: $9+4=13$

Point tally:

IV	V	VI
2	1	0

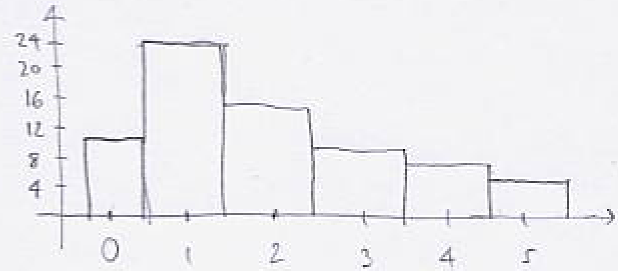
IV wins

b) Runoff btw IV and V
IV: $16+6=22$ wins
V: $18+3=21$

d) IV: $16\cdot 3+17\cdot 2+10\cdot 1=92$
V: $18\cdot 3+15\cdot 2+10\cdot 1=94$
VI: $9\cdot 3+11\cdot 2+23\cdot 1=72$
258

$43\cdot 6=258$ agree

4. (25pts) The number of movie theaters in cities with populations greater than 10,000 across a US state is shown in the table below.
a) Draw a histogram for the data.
b) Find the mode number of movie theaters.
c) Find the median number of movie theaters.
d) Find the mean number of movie theaters.
e) Find the standard deviation.



Movie Theaters	Frequency (cities)
0	11
1	24
2	15
3	9
4	7
5	5
	71

b) 1 is the mode

c) $\frac{71}{2} = 35.5 \rightarrow$ need the 36th number

$0, \ldots 0, 1, \ldots 1, 2, \ldots 2, 3, \ldots 3, 4, \ldots 4, 5, \ldots 5$
11th 12 35th 36th

median is 2.

d) $\frac{11 \cdot 0 + 24 \cdot 1 + 15 \cdot 2 + 9 \cdot 3 + 7 \cdot 4 + 5 \cdot 5}{71} = \frac{134}{71} = 1.887324$

$11 \cdot (0-1.88\ldots)^2 + 24 \cdot (1-1.88\ldots)^2 + \ldots + 5(5-1.88\ldots)^2 = 149.098592$

$s = \sqrt{\frac{149.098\ldots}{71-1}} = 1.459445$

5. (13pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

	Event	probability	odds against	odds in favor
a)	Drawing a queen from a deck of cards	$\frac{1}{13}$	12 : 1	1 : 12
b)	Getting exactly one head on three coin tosses	$\frac{3}{8}$	5 : 3	3 : 5
c)	Getting sum more than 8 on a roll of two dice	$\frac{5}{18}$	13 : 5	5 : 13

a) $\frac{4}{52} = \frac{1}{13}$

b) HHH
HHT
HTH
HTT ←
THH
THT ←
TTH ←
TTT

$\frac{3}{8}$

c) sum = 9 or above:

3,6	4,6	5,6	6,6
4,5	5,5	6,5	12
5,4	6,4	11	
6,3	10		
sum = 9			

$\frac{10}{36} = \frac{5}{18}$

6. (12pts) A game of chance is set up like this: the player pays \$10 and rolls a die. If the numbers 1 or 6 come up, the player wins \$21, if 4 comes up, the player wins \$15, otherwise the player wins nothing.
a) Find the expected value of this game.
b) What is the fair price of this game?
c) If a player played this game 100 times, how much would they expect to win or lose?

a) $E = \frac{2}{6} \cdot (21-10) + \frac{1}{6}(15-10) + \frac{3}{6}(-10) = \frac{22}{6} + \frac{5}{6} - \frac{30}{6} = -\frac{3}{6} = -\frac{1}{2}$

expected value $= -0.50$

b) fair price $= -0.50 + 10 = 9.50$

c) $100 \cdot (-0.50) = 50$

7. (5pts) In a subdivision of 43 houses, 13 have a pool, 21 have a three-car garage and 6 have both a pool and a three-car garage. If a home is randomly selected from the subdivision, what is the probability that it has a pool or a three-car garage?

P(pool or garage) = P(pool) + P(garage) − P(pool and garage)

$= \frac{13}{43} + \frac{21}{43} - \frac{6}{43} = \frac{28}{43}$

8. (10pts) The probability that a student gets a job within a year after graduating is 75%. Assuming that different students getting jobs are independent events. What is the probability that:
a) Two students will get jobs after graduating?
b) At least one from a group of three will not get a job after graduating?

a) P(1st gets job and 2nd gets job) = P(1st gets job) · P(2nd gets job)
$= 0.75 \cdot 0.75 = 0.5625$

b) P(at least one does not get job) = P(not (all three get jobs))
$= 1 -$ P(all three get jobs) $= 1 - 0.75 \cdot 0.75 \cdot 0.75 = 1 - 0.421875$
$= 0.578125$

9. (7pts) If $2,000 is deposited into an account bearing 2.55%, compounded daily, how much is in the account after two-and-a-half years?

$$A = 2000\left(1 + \frac{0.0255}{360}\right)^{360\cdot 2.5} = 2000(1.00007083)^{900}$$

$$= 2000 \cdot 1.06582\ldots = 2131.65$$

$$\left(A = P\left(1+\frac{r}{n}\right)^{nt}\right)$$

10. (14pts) The Swokowskis would like to save up for a luxury car.
a) How much should they deposit every quarter into an account with 3.75% interest, compounded quarterly, in order to have $35,000 in five years?
b) How much of the final amount is from deposits and how much from interest?

a) $$A = p\,\frac{\left(1+\frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}}$$

$$35000 = p \cdot \frac{\left(1+\frac{0.0375}{4}\right)^{4\cdot 5} - 1}{\frac{0.0375}{4}}$$

$$35000 = p \cdot 21.88\ldots \quad \Big| \div 21.88\ldots$$

$$p = \frac{35000}{21.88\ldots} = 1599.23$$

b) Amount from deposits

$$= 20 \cdot 1599.23 = 31,984.60$$

Amount from interest:

$$35000 - 31,984.60 = 3015.40$$

11. (16pts) The Middletons need to borrow \$650,000 to help cover the cost of the wedding of their daughter. Suppose they can get a 10-year loan with interest rate 6%, compounded monthly.
a) What is their monthly payment?
b) What is the balance on the loan after 8 years?

a) $P = m \dfrac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$

$650{,}000 = m \cdot \dfrac{1-\left(1+\frac{0.06}{12}\right)^{-12\cdot 10}}{\frac{0.06}{12}}$

$650{,}000 = m \cdot 90.073\ldots$

$m = \dfrac{650{,}000}{90.073} = 7216.33$

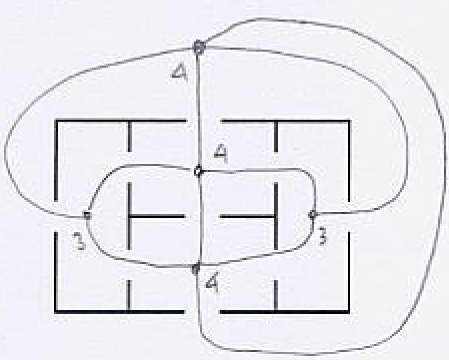
b) balance = present value of remaining payments

$P = 7216.33 \cdot \dfrac{1-\left(1+\frac{0.06}{12}\right)^{-12\cdot 2}}{\frac{0.06}{12}}$

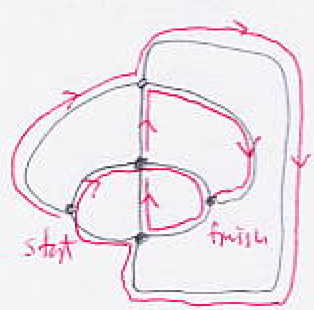
$= 7216.33 \cdot 22.56\ldots$

$= 162{,}821.09$

Bonus. (15pts) Below is a floor plan of a building, with doors joining rooms indicated.
a) Represent the floor plan as a graph (don't forget to include an "outside").
b) Use the graph to determine if it is it possible to walk around the building, passing through every door exactly once. If it is, draw the route.
c) Is it possible to do the same as in b), and start and finish outside?



b) The graph has 2 odd vertices, so it has an Euler path (starts and finishes at odd vertices)



c) No, since the graph does not have an Euler circuit