1. (26pts) Murray's Lady Gaga fan club wishes to put up a poster that highlights her best feature. To decide what this is, they survey the members to rank her following prominent features: clothes, hair, or nose.

Votes:	3	5	1	4	6	1	$\to$ 20 voters
1st	C	C	H	H	N	N	
2nd	H	N	C	N	C	H	
3rd	N	H	N	C	H	C	

a) Which choice wins the vote in a plurality election?
b) Which choice wins the vote in a plurality election with a runoff?
c) Which choice is the pairwise comparison winner?
d) Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

a) C: 3+5=8 wins
   H: 1+4= 5
   N: 6+1=7

b) C: 8+1= 9
   N: 7+4 = 11 wins

c) C; 8+6 = 14 wins
   H: 5+1 = 6

   C: 8+1 = 9
   N: 7+4 = 11 wins

   H: 5+3 =8
   N: 7+5 = 12 wins

   Overall   C: 1
   points   H: 0
             N: 2 wins

d) C: $8\cdot3 + 7\cdot2 + 5\cdot1 = 43$
   H: $5\cdot3 + 4\cdot2 + 11\cdot1 = 34$ } tie
   N: $7\cdot3 + 9\cdot2 + 4\cdot1 = 43$

   $\overline{120}$

   $20\cdot6 = 120$

2. (28pts) A town is trying to decide which community project to fund next. The choices are a fountain, a museum, a pool, or sidewalks. The preference rankings of the townspeople broke down into the following percentages.

Votes	18	15	9	17	11	16	14
1st	F	M	M	P	P	S	S
2nd	M	S	P	M	F	P	F
3rd	S	F	S	F	S	M	M
4th	P	P	F	S	M	F	P

a) Which choice wins the vote in a plurality election?
b) Which choice wins the vote in a plurality election with elimination?
c) Which choice wins the vote in a plurality election with a runoff? (Is it the same as b?)
d) Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

a) $F: 18 = 18$
$M: 15+9 = 24$
$P: 17+11 = 28$
$S: 16+14 = 30$ wins

b) Second round:

$M: 24+18 = 42$
$P: 28 \quad = 28 \leftarrow$ eliminated
$S: 30 \quad = 30$

Third round

$M \quad 42+17 = 59$ wins
$S \quad 30+11 = 41$

c) Runoff:
$P: 28+9 \quad = 37$
$S: 30+18+15 = 63$ wins

Winner not same as in b).

d) $F: 18 \cdot 4 + 25 \cdot 3 + 32 \cdot 2 + 25 \cdot 1 = 236$
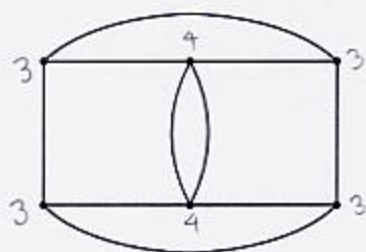$M: 24 \cdot 4 + 35 \cdot 3 + 30 \cdot 2 + 11 \cdot 1 = 272$ wins
$P: 28 \cdot 4 + 25 \cdot 3 + 0 \cdot 2 + 47 \cdot 1 = 234$
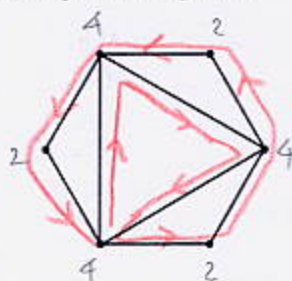$S: 30 \cdot 4 + 15 \cdot 3 + 38 \cdot 2 + 17 \cdot 1 = 258$

$\overline{ 1000}$

$100 \cdot (4+3+2+1) = 100 \cdot 10 = 1000$

3. (12pts) Determine whether each of the following graphs has an Euler path or an Euler circuit. If it does, find it, if not, explain why not.


Has more than 2 odd
vertices ⟹
no Euler path
no Euler circuit

All vertices even
⟹ Has Euler path,
which is a circuit

4. (15pts) Below is a floor plan of a building, with doors joining rooms indicated.
a) Represent the floor plan as a graph (don't forget to include an "outside").
b) Use the graph to determine if it is it possible to walk around the building, passing through every door exactly once. If it is, draw the route.
c) Is it possible to do the same as in b), and start and finish outside?
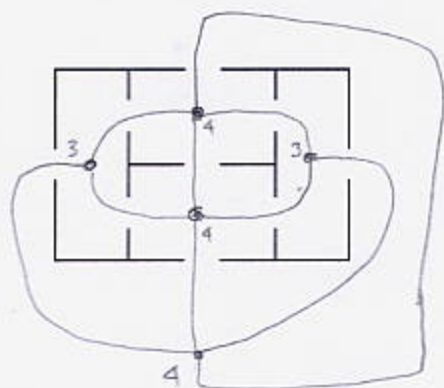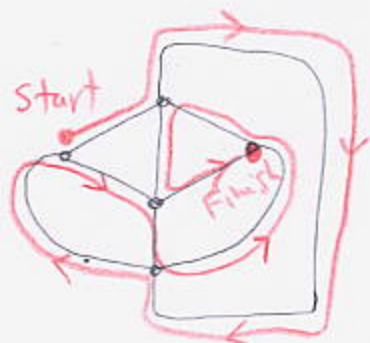

b) Has exactly two odd vertices

⟹ has an Euler path
but does not have an Euler circuit
Path starts and finishes at odd vertices

c) No, because this would mean
the existence of a circuit

5. (19pts) A weary tourist would like to visit Lexington, Louisville, Bowling Green and Paducah, while trying to minimize the distance traveled. The table below has the distances between the cities.
a) Draw a weighted graph that corresponds to this problem.
b) Use the brute force method to find the route that minimizes the distance traveled. First list all the possible orders of visits with Paducah the starting city.
c) Use the nearest neighbor algorithm to find an approximate solution to the problem. Is it the same as in c)?

	BG	Le	Lo
Le	154		
Lo	112	74	
P	151	262	220
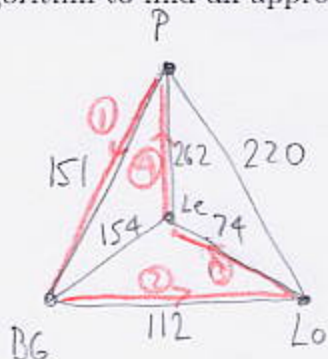
c) nearest neighbor:

$151 + 112 + 74 + 262 = 599$

is the same number.
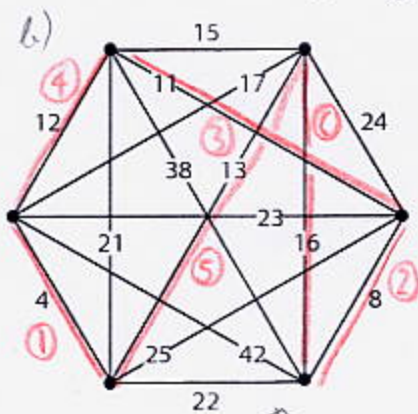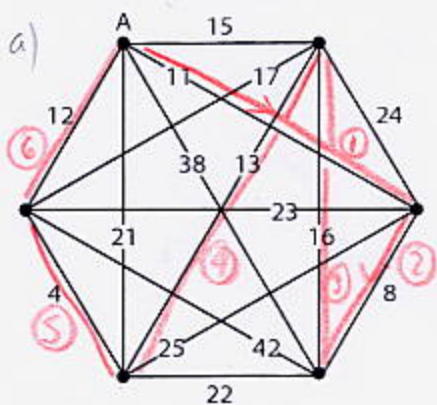
Brute force:

P BG Le Lo P: $151 + 154 + 74 + 220 = 599$
P BG Lo Le P: $151 + 112 + 74 + 262 = 599$
P Le BG Lo P: $262 + 154 + 112 + 220 = 748$
P Le Lo BG P: $262 + 74 + 112 + 151 = 599$ → shortest routes
P Lo BG Le P: $220 + 112 + 154 + 262 = 748$
P Lo Le BG P: $220 + 74 + 154 + 151 = 599$

Bonus. (10pts) Find an approximate solution to the traveling salesman problem. Show the weight of the found circuits. Use (one on each picture)
a) the nearest neighbor algorithm starting at A.
b) the greedy algorithm.

} they happen to give same circuit.

a)

b)

$11 + 8 + 16 + 13 + 4 + 12 = 64$

has same weight