Mathematical Concepts — Exam 3
MAT 117, Spring 2011 — D. Ivanšić

Name: Solution

Show all your work!

angle = (relative frequency) $\cdot$ 360° midrange = $\dfrac{\text{lowest value} + \text{highest value}}{2}$

$\overline{x} = \dfrac{x_1 + x_2 + \cdots + x_n}{n} = \dfrac{\sum_i x_i}{n}$ range = highest value − lowest value

$s = \sqrt{\dfrac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \cdots + (x_n - \overline{x})^2}{n-1}} = \sqrt{\dfrac{\sum_i (x_i - \overline{x})^2}{n-1}}$ $Z = \dfrac{X - \mu}{\sigma}$

1. (18pts) Over the course of a week, a curious observer counts the number of people sitting at the computers in the main hall of Murray State's library at 4PM. She gets the following numbers: 14, 23, 32, 21, 11, 8, 11.

a) Find the midrange.
b) Find the median.
c) Find the mean.
d) Find the range.
e) Find the standard deviation.

Order by size:
8, 11, 11, 14, 21, 23, 32

a) $\dfrac{8+32}{2} = 20$

b) median = 14
(middle number)

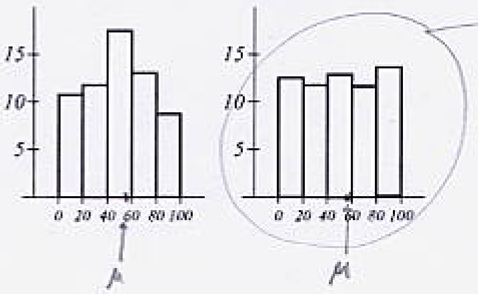
c) $\dfrac{8+11+11+14+21+23+32}{7} = \dfrac{120}{7} = 17.142857$

d) range = $32 - 8 = 24$

e) $\dfrac{(8-17.14)^2+(11-17.14)^2+\ldots+(32-17.14)^2}{7-1} = \dfrac{438.85\ldots}{6} = 73.14\ldots$

$s = \sqrt{73.14\ldots} = 8.552360$

2. (6pts) Histograms for two data sets, which have the same mean $\mu = 59$, are shown. Which of the data sets will have a greater standard deviation and why?

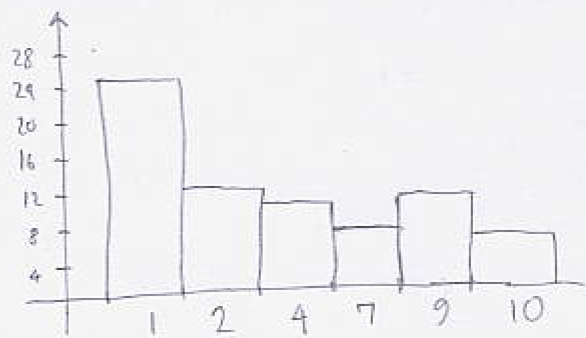


has greater standard deviation, since data is more spread out away from the mean

3. (28pts) A thrift store has a selection of cheaply priced items. The table below indicates the prices that appear in the store and how many items have them.

a) Draw a histogram for the data.
b) Find the mode price.
c) Find the median price.
d) Find the mean price.
e) Find the standard deviation.

a)



Price	Frequency
\$1	25
\$2	13
\$4	11
\$7	8
\$9	12
\$10	7
	76

b) \$1, the most frequently occuring price

c) 1, ..., 1, 2, ..., 2, 4, ..., 4, 7, ..., 7, 9, ..., 9, 10, ..., 10

25th 38th 39th

$76/2 = 38$ → Need 38th and 39th

$\text{median} = \frac{2+4}{2} = 3$

d) $\bar{x} = \dfrac{25\cdot 1 + 13\cdot 2 + 11\cdot 4 + 8\cdot 7 + 12\cdot 9 + 7\cdot 10}{76} = \dfrac{329}{76} = 4.328947$

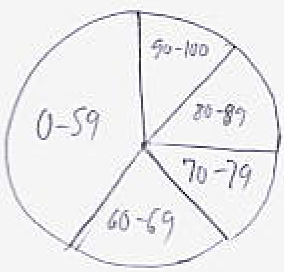
e) $\dfrac{25\cdot(1-4.32..)^2 + 13\cdot(2-4.32..)^2 + \dots + 7\cdot(10-4.32..)^2}{76-1} = \dfrac{892.77..}{75} = 11.903$

$s = \sqrt{11.903..} = 3.450172$

4. (19pts) The distribution of scores on our exam 2 is shown below. Do the following:
a) Find the relative frequencies.
b) Draw a pie chart (find angles first).
c) Enter a representative value for each interval.
d) Use the representative values to estimate the mean of data. How does it compare to the actual mean of 65.1?

Range of Scores	Number of Students	Relative Frequency	Angle	Representative Value
90–100	5	5/39 = 0.1281	46	95 ← $\frac{90+100}{2}$
80–89	5	5/39 ≈ 0.1281	46	84.5
70–79	6	6/39 = 0.1539	55	74.5
60–69	7	7/39 = 0.1795	65	64.5
0–59	16	16/39 = 0.4103	148	29.5
	39			

b)



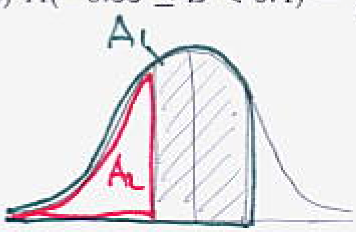
d) $\frac{5\cdot 95 + 5\cdot 84.5 + 6\cdot 74.5 + 7\cdot 64.5 + 16\cdot 29.5}{39}$

$= \frac{2268}{39} = 58.153846$ ← It's quite far from 65.1, because we used 29.5 as a rep. value for the range 0–59. Most students in this range did much better than 30.

5. (13pts) Compute the following areas under a standard normal distribution curve. Draw a picture showing which area you are computing.

a) $A(-0.33 \leq Z < 0.4) = A_1 - A_2 = 0.6554 - 0.3707$

$= 0.2847$



−0.33 0 0.4

b) $A(Z > -0.65) = 1 - A_1 = 1 - 0.2578$

$= 0.7422$

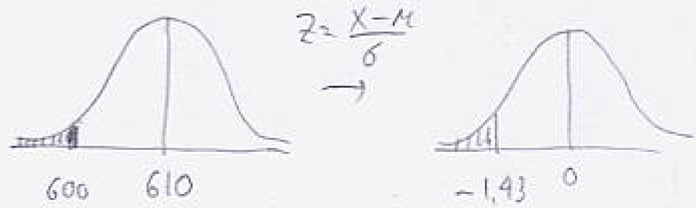
−0.65 0

6. (16pts) The weight of a box of chocolate-milk powder is normally distributed with mean 610 grams and standard deviation 7 grams. The declared weight on the package is 600 grams. Draw a picture showing which area you are computing as you answer:

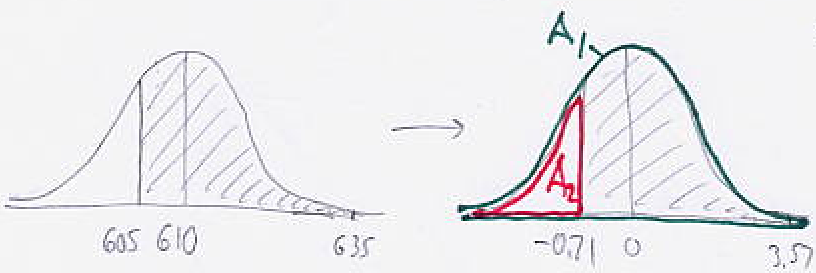
a) What percentage of boxes have weight lower than declared weight?

b) What percentage of boxes have weight between 605 and 635 grams?

a) $P(X < 600) = A\left(Z < \frac{600-610}{7}\right) = A(Z < -1.43) = 0.0764$



b) $P(605 \leq X \leq 635) = A\left(\frac{605-610}{7} \leq Z \leq \frac{635-610}{7}\right) = A(-0.71 \leq Z \leq 3.57)$

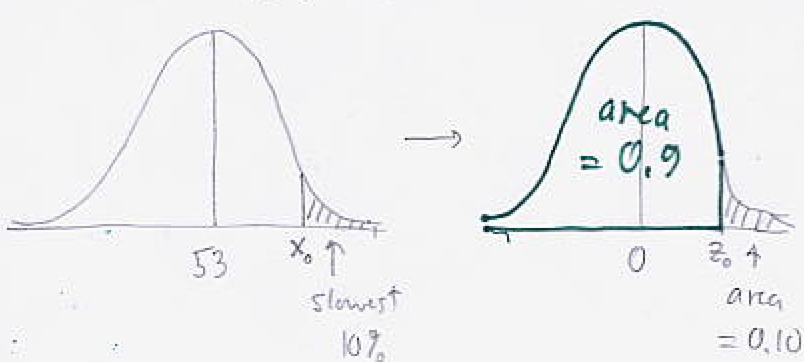


$= A_1 - A_2$

$= 1 - 0.2389$

$= 0.7611$

Bonus. (10pts) Over the course of many years, the organizers of the 10K race "Find Your Fast" have noticed that participants' running times are normally distributed with mean 53 minutes and standard deviation 7 minutes. As the number of participants has risen, the organizers have decided to make the race more competitive by demanding that an applicant's time on some 10K race they have run in the past be better than the times of the slowest 10% of the "Find Your Fast" participants. What is the highest qualifying time for the race? *(Hint: this problem is the reverse of what we usually do with a normal distribution. Here, the area is given: you have to find the number on the axis that this area corresponds to. You will also need to apply conversion to the standard normal distribution.)*



Find z_0 so corresponding area is 0.9

- look up which area in table is closest to 0.9 ← 0.8997, corresponding to $z_0 = 1.28$

Now, convert back to x_0!

$\frac{x_0 - 53}{7} = 1.28 \quad | \cdot 7$

$x_0 - 53 = 8.96 \quad | + 53$

$x_0 = 61.96$

An applicant has to run a time better than 62 min to qualify for the race.