Mathematical Concepts — Exam 2
MAT 117, Spring 2011 — D. Ivanšić

Name: Solution

Show all your work!

$\frac{a}{b} = \frac{1-P(E)}{P(E)}$ $\quad P(E) = \frac{b}{a+b}$ where odds against E are $a:b$ $\quad P(B|A) = \frac{n(A \text{ and } B)}{n(A)}$
$P(A \text{ or } B) = P(A)+P(B)-P(A \text{ and } B) = P(A)+P(B)$ (if A and B are mutually exclusive)
$P(A \text{ and } B) = P(A) \cdot P(B|A) \quad P(A \text{ and } B) = P(A) \cdot P(B)$ if A and B are independent
$E = P_1 \cdot A_1 + P_2 \cdot A_2 + \cdots + P_n \cdot A_n$

1. (8pts) The number and type of trees on a college campus are as follows:

Tree	oak	chestnut	cherry	magnolia
Number	31	22	25	12

total $= 90$

A couple is looking to nail their shoes to a random tree on campus. What is the probability that the chosen tree is
a) a chestnut?
b) a cherry or an oak?
c) not a magnolia?

a) $P(\text{chestnut}) = \frac{22}{90} = \frac{11}{45} = 0.244444$

$P(\text{cherry or oak}) = \frac{25+31}{90} = \frac{56}{90} = \frac{28}{45} = 0.622222$

$P(\text{not magnolia}) = 1 - P(\text{magnolia}) = 1 - \frac{12}{90} = \frac{90}{90} - \frac{12}{90} = \frac{78}{90} = \frac{13}{15}$
$= 0.866667$

2. (18pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

	Event	probability	odds against	odds in favor
a)	Getting heads on a coin toss	$\frac{1}{2}$	1:1	1:1
b)	Drawing a black ace or a jack from a deck of cards	$\frac{6}{52} = \frac{3}{26}$	23:3	3:23
c)	Getting exactly one head on three coin tosses	$\frac{3}{8}$	5:3	3:5
d)	Getting sum less than 5 on a roll of two dice	$\frac{6}{36} = \frac{1}{6}$	5:1	1:5
e)	Rolling a 5 and a 4 (not sum!) on a roll of two dice	$\frac{2}{36} = \frac{1}{18}$	17:1	1:17

b) black aces: 2
jacks: 4
6

c) HHH
HHT
HTH
HTT
THH
THT
TTH
TTT
8

exactly one head
3 times

d) 1+1
1+2
1+3
2+1
2+2
3+1
6

e) 5,4
4,5
2

3. (14pts) A game of chance is set up like this: the player pays \$2 and tosses a raisin into a box with hamsters Gabe, Miles and Snoopy which scramble and get the raisin with likelihoods 30%, 10% and 50%, respectively. 10% of the time the hamsters ignore the raisin. Depending on whether Gabe, Miles or Snoopy get the raisin, the player wins \$3, \$5 or \$1, respectively, otherwise the player wins nothing.

a) Find the expected value of this game.
b) What is the fair price of this game?
c) If a player played this game 200 times, how much would they expect to win or lose?

a) $E = P(\text{Gabe})\cdot(3-2) + P(\text{Miles})\cdot(5-2) + P(\text{Snoopy})\cdot(1-2) + P(\text{none})\cdot(-2)$
$= 0.3\cdot 1 + 0.1\cdot 3 + 0.5\cdot(-1) + 0.1\cdot(-2)$
$= 0.3 + 0.3 - 0.5 - 0.2 = -0.10$ expect to lose 10c per game

b) Fair price = E + price = $-0.10 + 2 = \$1.90$

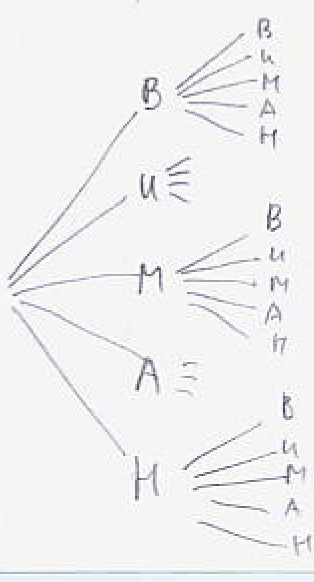
c) $200\cdot(-0.10) = -20$ The player would expect to lose about \$20

4. (18pts) A bag contains five tiles with the letters B, U, M, A, H . A tile is drawn from the bag and then replaced. This is done twice.

a) Determine the number of points in the sample space.
b) Construct a tree diagram and list the sample space. (You may do these partially.)

Determine the probability that:
c) exactly one of the letters is a U.
d) both letters are consonants.
e) both letters are vowels.
f) at least one letter is B or M.

a) $5\cdot 5 = 25$



B,B
B,U
B,M
B,A
B,H
U,B
U,U
U,M
U,A
U,H
⋮
H,B
H,U
H,M
H,A
H,H

c) $U\_ + \_U = 8$ choices (4 choices) $P(\text{exactly one U}) = \frac{8}{25} = 0.32$

d) $B\_ + M\_ + H\_ = 9$ choices (3 choices each) $P(\text{both consonants}) = \frac{9}{25} = 0.36$

e) $U\_ + A\_ = 4$ choices (2 choices each) $P(\text{both vowels}) = \frac{4}{25} = 0.16$

f) $P(\text{neither is B or M}) = \frac{9}{25}$ $U\_ + A\_ + H\_ = 9$ (3 choices)
$P(\text{at least one is B or M}) = 1 - \frac{9}{25} = \frac{25}{25} - \frac{9}{25} = \frac{16}{25} = 0.64$

5. (10pts) The manager of a car repair shop notices that the probability that a car brought into the shop requires an oil change is 0.6. The probability that a car needs brake repair is 0.4, and the probability that it needs both brake repair and an oil change is 0.2. What is the probability that:
a) a car brought in the shop needs an oil change or brake repair?
b) a car brought in the shop needs neither an oil change nor brake repair?

a) P(oil or brake) = P(oil) + P(brake) − P(oil and brake)
$= 0.6 + 0.4 - 0.2 = 1 - 0.2 = 0.8$

b) P(neither oil nor brake) = P(NOT (oil or brake))
= 1 − P(oil or brake)
$= 1 - 0.8 = 0.2$

6. (14pts) The probability that you can obtain a special kind of whole-wheat bread at a grocery store in the afternoon is 70% (it often sells out). Assume that availability of this bread on different days are independent events. What is the probability that:
a) on two afternoon trips to the store you find the bread both times?
b) on three afternoon trips to the store, you find the bread at least once?
c) on three afternoon trips to the store, you find the bread the first and the third time, but not the second?

a) P(1st and 2nd) = P(1st) · P(2nd) $= 0.7 \cdot 0.7 = 0.49$

b) P(at least once) = 1 − P(never) = 1 − P(not 1st and not 2nd and not 3rd)
= 1 − P(not 1st) · P(not 2nd) · P(not 3rd) $= 1 - 0.3 \cdot 0.3 \cdot 0.3 = 1 - 0.3^3 = 0.973$

c) P(1st and not 2nd and 3rd) = P(1st) · P(not 2nd) · P(3rd) $= 0.7 \cdot 0.3 \cdot 0.7 = 0.147$

7. (18pts) From a group of 11 men and 17 women, two people are chosen at random. What is the probability that:
a) both are men?
b) at least one is a man?
c) exactly one person is a woman?

28 total

a) $P\left(\begin{smallmatrix}\text{1st}\\\text{man}\end{smallmatrix} \text{ and } \begin{smallmatrix}\text{2nd}\\\text{man}\end{smallmatrix}\right) = P(\text{1st man}) \cdot P\left(\begin{smallmatrix}\text{2nd}\\\text{man}\end{smallmatrix} \middle| \begin{smallmatrix}\text{1st}\\\text{man}\end{smallmatrix}\right) = \frac{11}{\cancel{28}_{14}} \cdot \frac{\cancel{10}^{5}}{27} = \frac{55}{378} = 0.145503$

b) $P(\text{at least one man}) = 1 - P(\text{neither is man}) = 1 - P\left(\begin{smallmatrix}\text{1st}\\\text{woman}\end{smallmatrix} \text{ and } \begin{smallmatrix}\text{2nd}\\\text{woman}\end{smallmatrix}\right)$

$= 1 - P\left(\begin{smallmatrix}\text{1st}\\\text{woman}\end{smallmatrix}\right) P\left(\begin{smallmatrix}\text{2nd}\\\text{woman}\end{smallmatrix} \middle| \begin{smallmatrix}\text{1st}\\\text{woman}\end{smallmatrix}\right)$

$= 1 - \frac{17}{\cancel{28}_{7}} \cdot \frac{\cancel{16}^{4}}{27} = 1 - \frac{68}{189} = \frac{189}{189} - \frac{68}{189} = \frac{121}{189} = 0.640212$

c) $P(\text{exactly one woman}) = P\left(\begin{smallmatrix}\text{1st}\\\text{woman}\end{smallmatrix} \text{ and } \begin{smallmatrix}\text{2nd}\\\text{man}\end{smallmatrix}\right) + P\left(\begin{smallmatrix}\text{1st}\\\text{man}\end{smallmatrix} \text{ and } \begin{smallmatrix}\text{2nd}\\\text{woman}\end{smallmatrix}\right)$

$= P\left(\begin{smallmatrix}\text{1st}\\\text{woman}\end{smallmatrix}\right) \cdot P\left(\begin{smallmatrix}\text{2nd}\\\text{man}\end{smallmatrix} \middle| \begin{smallmatrix}\text{1st}\\\text{woman}\end{smallmatrix}\right) + P\left(\begin{smallmatrix}\text{1st}\\\text{man}\end{smallmatrix}\right) \cdot P\left(\begin{smallmatrix}\text{2nd}\\\text{woman}\end{smallmatrix} \middle| \begin{smallmatrix}\text{1st}\\\text{man}\end{smallmatrix}\right)$

$= \frac{17}{28} \cdot \frac{11}{27} + \frac{11}{28} \cdot \frac{17}{27} = \frac{\cancel{2} \cdot 11 \cdot 17}{\cancel{28}_{14} \cdot 27} = \frac{187}{378} = 0.494709$

Bonus. (10pts) Suppose a precision-guided bomb hits a target with probability 0.8. A bridge is targeted by two bombs: whether one hits the target is independent of whether the other one hits. If a single bomb hits the bridge, it collapses with probability 0.3, and if both bombs hit, it collapses with probability 0.9. What is the probability that the bridge collapses after an attack by two bombs? *Hint: P(collapse)=P(exactly one bomb hits AND collapses)+ P(both bombs hit AND collapses)*

$= P\left(\begin{smallmatrix}\text{exactly}\\\text{one hits}\end{smallmatrix}\right) \cdot \underbrace{P\left(\text{collapses} \middle| \begin{smallmatrix}\text{exactly}\\\text{one hits}\end{smallmatrix}\right)}_{0.3} + P\left(\begin{smallmatrix}\text{both}\\\text{hit}\end{smallmatrix}\right) \cdot \underbrace{P\left(\text{collapse} \middle| \begin{smallmatrix}\text{both}\\\text{hit}\end{smallmatrix}\right)}_{0.9}$

$= P\Big(\underbrace{(\text{1st hits and 2nd misses})}_{\text{mutually}} \text{ OR } \underbrace{(\text{1st misses and 2nd hits})}_{\text{exclusive}}\Big) \cdot 0.3 + P(\text{1st hits and 2nd hits}) \cdot 0.9$

$= \Big(P(\text{1st hits}) \cdot P(\text{2nd misses}) + P(\text{1st misses}) \cdot P(\text{2nd hits})\Big) \cdot 0.3 + P(\text{1st hits}) \cdot P(\text{2nd hits}) \cdot 0.9$

$= (0.8 \cdot 0.2 + 0.2 \cdot 0.8) \cdot 0.3 + 0.8 \cdot 0.8 \cdot 0.9$

$= 0.32 \cdot 0.3 + 0.64 \cdot 0.9 = 0.096 + 0.576 = 0.672$