Mathematical Concepts — Exam 1
MAT 117, Spring 2011 — D. Ivanšić

Name: Solution

Show all your work!

$i = prt$ $A = p(1+rt)$ $A = p\left(1+\frac{r}{n}\right)^{nt}$ $A = p\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$ $P = m\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$ $APY = \left(1+\frac{r}{n}\right)^{n}-1$

1. (8pts) Sharon would like to buy a dinner table in the near future. How much should she deposit now, in an account bearing 5.4% simple interest, in order to have \$1500 in seven months?

$A = p(1+rt)$

$1500 = p\left(1+0.054\cdot\frac{7}{12}\right)$

$1500 = p\cdot 1.0315$

$p = \frac{1500}{1.0315} = 1454.19$

2. (10pts) You borrowed \$300 from an uncle, and repaid him in 6 months with \$314. What simple annual interest rate did you pay on this loan?

$i = 314-300$ $i = prt$

$14 = 300\cdot r\cdot\frac{6}{12}$

$14 = 150r$

$r = \frac{14}{150} = 0.0933..$

$r = 9.33\%$

3. (8pts) William Walker deposits \$30,000 into an account bearing 4% interest, compounded weekly. How much does he have in 3 years?

$A = p\left(1+\frac{r}{n}\right)^{nt}$

$A = 30{,}000\left(1+\frac{0.04}{52}\right)^{52\cdot 3}$ (156) $= 30000\cdot 1.1274..$

$= 33{,}823.35$

4. (14pts) On March 3rd, Ignacio paid \$890 for a spring break trip . He put 15% down, and the rest he financed with a 90-day loan with a simple interest rate of 7%.
a) When is the loan due?
b) If Ignacio makes a partial payment of \$200 on April 20th, how much does he owe on the due date?

Financed 85% of 890, which is $0.85 \cdot 890 = \$756.50$

a) Approx June 3rd

Mar 28 = 31 − 3
Apr 30
May 31
June 3
92

June 3rd is 92 days
June 1st is 90 days

b) Interest from Mar 3rd to Apr. 20th, 48 days

Mar 28
Apr 20
48

$i = 756.50 \cdot 0.07 \cdot \frac{48}{360} = 7.06$

$200 - 7.06 = 192.94$ toward principal.

$756.50 - 192.94 = 563.56$ new principal

Interest from Apr 20 to June 1st (42 days = 90 − 48)

$i = 563.56 \cdot 0.07 \cdot \frac{42}{360} = 4.60$

Due on June 1st: $563.56 + 4.60 = 568.16$

5. (14pts) The Zhangs would like to save up to add a pool to their home.
a) How much should they deposit every quarter into an account with 3.5% interest, compounded quarterly, in order to have \$25,000 in five years?
b) How much of the final amount is from deposits and how much from interest?

a) $A = P \dfrac{\left(1+\frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}}$

$25000 = P \dfrac{\left(1+\frac{0.035}{4}\right)^{4 \cdot 5} - 1}{\frac{0.035}{4}}$

$25000 = P \cdot 21.753$

$P = \dfrac{25000}{21.753} = 1149.26$

b) Total deposits:

$1149.26 \cdot 4 \cdot 5 = 22{,}985.20$

(4: quarterly; 5: years)

From interest: $25{,}000 - 22{,}985.20 = 2{,}014.80$

6. (32pts) Wall-E, the trash-collecting robot, wishes to have more room to store his scavenged items and decides to have a warehouse built at a cost of \$430,000. He finances this with a 4.65% mortgage, compounded monthly, over 30 years.

a) What is his montly payment on the loan?

b) What are his total payments over the course of the loan? How much of this amount is for interest?

c) How much does he owe after 10 years?

d) How much of his 121st payment (it's the next one after 10 years) goes toward interest, and how much towards the principal?

a) $P = m \dfrac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$

$430{,}000 = m \dfrac{1-\left(1+\frac{0.0465}{12}\right)^{-12\cdot 30}}{\frac{0.0465}{12}}$

$430{,}000 = m \cdot 193.93\ldots$

$m = \dfrac{430{,}000}{193.93\ldots} = 2217.24$

b) Total payments:

$2217.24 \cdot 12 \cdot 30 = 798{,}206.40$

Interest is $798{,}206.40 - 430{,}000.00 = 368{,}206.40$

c) Amount owed = present value of remaining payments

10 years = 120 payments,

$360 - 120 = 240$ remaining

$P = 2217.24 \cdot \dfrac{1-\left(1+\frac{0.0465}{12}\right)^{-240}}{\frac{0.0465}{12}}$

$= 2217.24 \cdot 156{,}061\ldots$

$= 346{,}024.79$

d) Interest on 121st payment:

$i = 346{,}024.79 \cdot \dfrac{0.0465}{12} = 1340.85$

Toward principal:

$2217.24 - 1340.85 = 876.39$

7. (14pts) Betty decided to buy new furniture for her home, for which she will need \$12,000. If she can deposit \$200 monthly into an account bearing 8.25%, compounded monthly, how long will it take until she has the desired amount?

$$A = P\,\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$$

$$12{,}000 = 200 \cdot \frac{\left(1+\frac{0.0825}{12}\right)^{12t}-1}{\frac{0.0825}{12}} \quad | \div 200$$

$$60 = \frac{(1.006875)^{12t}-1}{0.006875} \quad | \cdot 0.006875$$

$$0.4125 = (1.006875)^{12t} - 1 \quad | +1$$

$$1.4125 = 1.006875^{12t} \quad | \log$$

$$\log 1.4125 = \log 1.006875^{12t}$$

$$\log 1.4125 = 12t \log 1.006875$$

$$t = \frac{\log 1.4125}{12 \log 1.006875} = 4.200\ldots$$

$$t \approx 4.2 \text{ years}$$

Bonus. (10pts) George deposits \$135 every quarter into an account earning 3.4% interest, compounded quarterly. He does this for three years, then stops making deposits, leaving the accumulated money in the account. How much does he have five years after he started making deposits?

After three years, amount in the account is determined using the annuity formula

$$A = P\,\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$$

$$A = 135 \cdot \frac{\left(1+\frac{0.034}{4}\right)^{4\cdot 3}-1}{\frac{0.034}{4}}$$

$$= 135 \cdot 12.577\ldots$$

$$= 1697.92$$

Now we treat the 1697.92 as a one-time deposit left for two years:

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$= 1697.92\left(1+\frac{0.034}{4}\right)^{4\cdot 2}$$

$$= 1697.92 \cdot 1.070057\ldots$$

$$= 1816.87$$