Mathematical Reasoning — Handout
MAT 312, Fall 2011 — D. Ivanšić

Review Sheet

Sections 5.1, 5.2, 6.1–6.3

5.1 Definition of an inductive set
Know principle of mathematical induction
Prove various statements using induction, like in homework

5.2 Know other forms of mathematical induction:
basis step may be a number other than 1
assumption of induction step is that statement is valid for numbers $1, 2, \ldots, k$
Prove various statements using induction, like in homework

6.1 Definition of function, domain, codomain, range
Be able to find range, or set up a domain or codomain of a function
Be able to draw arrow diagrams
Definition of preimage and finding the preimage of an element
Be familiar with examples of functions
$\mathbf{R} \to \mathbf{R}$, $\mathbf{Z} \to \mathbf{Z}$, $\mathbf{Z}_n \to \mathbf{Z}_n$, $\mathbf{R} \times \mathbf{R} \to \mathbf{R}$, $\mathbf{Z} \times \mathbf{Z} \to \mathbf{Z}$, etc.
The number of divisors function

6.2 Definition of equality of functions
Functions involving congruences
Sequences as functions $\mathbf{N} \to \mathbf{R}$

6.3 Definitions of injection, surjection, bijection
Negating definitions of injection, surjection, bijection
Proving that a function is injective
The horizontal line test for injectivity of functions $\mathbf{R} \to \mathbf{R}$
Proving that a function is surjective (closely connected to finding the range)
Proving that a function is bijective
Altering domain or codomain to make the function injective, surjective bijective.

Sections 3.1–3.5, 4.1–4.3

3.1 Divisibility and congruence (definitions and manipulating statements)

3.2 Proving a statement $P \Longrightarrow Q$ by proving the contrapositive $\neg Q \Longrightarrow \neg P$
Proving biconditional statements (prove both directions)
Proofs by construction (explicitly producing the object whose existence is claimed)
Proofs without construction (showing an object exists, without knowing what it is)

3.3 Proofs by contradiction
Know to use contrapositive instead of contradiction when convenient
Avoid contradiction when a proof can be done directly (often with inequalities)

3.4 Proofs that are broken up into cases by, for example:
- odd and even integers
- remainders when divided by a certain number
- positive and negative real numbers, etc.

Basic properties of absolute value and triangle inequality (3.23 and 3.25)

3.5 Division algorithm and congruences
Know how to manipulate congruences:
- congruences may be added
- congruences may be multiplied
- congruences may be raised to a power

Applications to problems concerning divisibility (often times by contrapositive)

4.1 Definitions of $A \cap B$, $A \cup B$, $A - B$, A^c
Negating definitions of $A \cap B$, $A \cup B$, $A - B$, A^c
Finding intersections, unions, differences of given sets
Drawing Venn diagrams

4.2 Proving $A \subseteq B$ using the "choose an element method"
Checking set relationships by drawing Venn diagrams

4.3 Know all basic formulas on handout for operations on sets
Make connection between basic formulas for set relationships
and basic formulas for logical equivalences
Determining set equalities using established set formulas

Sections 1.1, 1.2, 2.1–2.4

1.1	Sentences and statements (truth value of) Conditional Statements Closure properties of number systems Understanding when the statement "If P, then Q" is true
1.2	Definition of even and odd integer Know-show table Proof-writing guidelines (read again) Constructing simple proofs involving integers
2.1	Truth tables for $\neg P$, $P \wedge Q$, $P \vee Q$, $P \Longrightarrow Q$ Showing equivalence using truth tables Other forms of the conditional statement (language) Biconditional statement $P \Longleftrightarrow Q$
2.2	Converse and contrapositive of a statement Theorem 2.9: established logical equivalences, know all except last two Negating statements in words Determining logical equivalences using established ones
2.3	Sets, set notation, roster method, set builder notation Predicates, their truth sets, and finding them Quantifiers Turning statements with $\forall$, $\exists$ into English and vice-versa
2.4	Negations of quantified statements with one or more quantifiers Converting statements with more than one quantifier from symbols to English and vice-versa.