1. (5pts) Set up the integral for the area of the region that is between $y = x^2 - 2x$ and $y = 4 - x^2$. Sketch a picture. Do not evaluate the integral.

2. (8pts) Sketch the region enclosed by the curves $y = e^x$, $y = \frac{1}{2}$ and $x = 1$. Find the volume of the solid obtained by rotating this region about the x-axis. Sketch the solid and a typical cross-section or cylindrical shell, depending on the method you are using.

3. (5pts) Find the solution of the initial-value problem $x - 2y(x^2 + 1)y' = 0$, $y(1) = 0$.

Evaluate the following integrals:

4. (6pts) $\int x^2(\ln x)^2\,dx =$

5. (8pts) $\int \frac{\sqrt{x^2-9}}{x^4}\,dx =$

6. (6pts) Sketch the graph of the function $r = \sin 2\theta$ in cartesian coordinates. Then use the intervals of increase and decrease of that graph to help you sketch the polar curve $r = \sin 2\theta$. Indicate which piece of the cartesian graph corresponds to which piece of the polar graph.

7. (6pts) Find the area enclosed by the polar curve $r = \sin 2\theta$ from the previous problem.

8. (8pts) Find the interval and radius of convergence and for the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^2}(x-1)^n$. Don't forget to check the endpoints.

9. (7pts) The population of Sprout City that started with 10,000 residents is growing exponentially so that in two years it is 12,036.
a) Write the differential equation that models the growth of the population.
b) Find the formula for the population after t years.
c) How long does it take until the population is double the starting population?

10. (7pts) Determine whether each of the following series converges. State which test you used.

a) $\displaystyle\sum_{n=2}^{\infty} \frac{n}{n^5 - 1}$

b) $\displaystyle\sum_{n=0}^{\infty} \frac{3}{5 + 2^n}$

The following two problems are about the integral $I = \int_0^1 \cos x^2 \, dx$. It is impossible to find the antiderivative of $\cos x^2$ in terms of elementary functions, so we have to resort to a numerical approximation.

11. (6pts) Use the midpoint method to approximate I with accuracy 10^{-4}:
a) Find the maximum value of $|f''(x)|$ on the interval $[0, 1]$ (use the graph on your calculator).
b) Use the error estimate $|E_M| \leq \frac{K(b-a)^3}{24n^2}$ to find how many subintervals you need.
c) Approximate I using the midpoint rule for the number of subintervals you found in b).

12. (6pts) Now approximate I with accuracy 10^{-4} using series:
a) Obtain the Maclaurin series for $\cos x^2$ from the known Maclaurin series for $\cos x$.
b) Integrate the Maclaurin series for $\cos x^2$ and evaluate it in the required bounds.
c) Find the sum of the series in b) with accuracy 10^{-4}.

13. (2pts) Compare results of the previous two problems. What is the farthest apart that your answers may be? Which method is easier if you had to perform all the calculations by hand?

Bonus (8pts) The power series $\sum_{n=1}^{\infty}(-1)^n \frac{2^{n-3}x^n}{5^{2n+1}}$ is given.

a) What function is represented by this power series?
b) What is the radius of convergence of the power series?
c) What problem do you run into if you try to find the power series for the function using the formula for the coefficients of the Taylor series?