Spring '03/MAT 308/Exam 6 **Name:** *Show all your work.*

The rules: you may use your book and notes on this take-home exam. Your work is to be entirely your own: you may not talk to anybody else about the exam problems. Turn the exam in by 4PM Monday.

1. (6pts) Compute $\lim_{x\to 0} \dfrac{e^x - 1 - x}{x^2}$ by using the Maclaurin expansion of e^x. Verify your answer by L'Hospital's rule.

2. (4pts) Use the MacLaurin series for $\sin x$ and $\cos x$ to show that $\dfrac{d}{dx} \sin x = \cos x$.

3. (3pts) Use a familiar power series expansion to find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{3^{2n+1}(2n+1)!}$.

4. (6pts) Suppose we know that $\sum c_n x^n$ converges for $x = 4$ and diverges for $x = 5$.
a) What can you say about the convergence of $\sum c_n x^n$ for $x = -1$ and $x = 7$? Explain.
b) What can you say about the convergence of $\sum n c_n x^{n-1}$ for $x = -1$ and $x = 7$? Explain.

5. (8pts) Find the interval and radius of convergence and for the series $\sum_{n=0}^{\infty} \frac{n}{3^{2n}}(x+2)^n$. Don't forget to check the endpoints.

6. (8pts) Use the Maclaurin series for e^x to obtain an estimate of $\frac{1}{e}$ with accuracy 10^{-4}.

7. (8pts) Use integration, differentiation and manipulation of a familiar power series to obtain the Maclaurin series for $f(x) = \ln(4 - x)$. State the radius of convergence.

8. (7pts) Find the Taylor series expansion of $f(x) = e^{2x}$ about $a = 3$. (Use the the general formula for a Taylor series.)

Bonus (5pts) Do the above by manipulating the Maclaurin series for e^x instead of following the general formula for a Taylor series.