Spring '03/MAT 308/Exam 4 **Name:** *Show all your work.*

1. (4pts) Sketch the points in the plane with polar coordinates:

$$\left(1, \tfrac{2\pi}{3}\right) \qquad \left(-2, \tfrac{\pi}{2}\right) \qquad \left(-3, -\tfrac{7\pi}{6}\right)$$

2. (5pts) Find the equation of the tangent line to the parametric curve $x = 1 - t^3$, $y = t^2 + 2t$ at the point $(0, 3)$.

3. (5pts) Describe the motion of a particle with position (x, y) as t varies in the given interval. Eliminate the parameter, identify the curve along which the particle moves and sketch the motion of the particle.

$$x = \sin t$$
$$y = \cos^2 t$$
$$0 \leq t \leq \pi$$

4. (4pts) Find all possible polar coordinates of the point whose cartesian coordinates are $(-2, -2)$. A picture will help you.

5. (6pts) Sketch the graph of the function $r = 3(1 + \sin\theta)$ in cartesian coordinates. Then use the intervals of increase and decrease of that graph to help you sketch the polar curve $r = 3(1 + \sin\theta)$. Indicate which piece of the cartesian graph corresponds to which piece of the polar graph.

6. (6pts) Find the length of the parametric curve given by $x = e^t - t$, $y = 4e^{t/2}$, $0 \leq t \leq 2$.

7. (10pts) Use intervals of increase and decrease of the functions $x(t)$ and $y(t)$ to sketch the parametric curve $x = t^3 - 3t^2$, $y = t^3 - 3t$. (You must show your analysis - just a calculator-generated picture will not do.)

8. (10pts) Sketch a picture and find the area of the region that is inside the polar curve $r = -3\cos\theta$ and outside the polar curve $r = 2 + \cos\theta$.

Bonus. (5pts) Use the cartesian graph of the function to help you sketch the polar curve $r = \tan\theta$. This curve has two asymptotes. What are their equations? Use a parametric description of the curve to explain the equation of the asymptotes.