Spring '03/MAT 308/Exam 3 **Name:** *Show all your work.*

1. (5pts) Use the direction field below to roughly sketch the graphs of the solutions of the equation $y' = F(x, y)$ with the initial conditions:

a) $y(1) = 2.5$

b) $y(-1) = 1$

2. (5pts) Sketch the direction field for the differential equation $y' = \frac{1}{4}(y-1)^2$. Sample at the integer-valued coordinates between -1 and 2 for both x and y. If you have the graph of one solution of the differential equation, how are the graphs of the other solutions related to it?

3. (5pts) Show that the function $y = \dfrac{2 + \ln x}{x}$ is the solution to the initial-value problem $x^2y' + xy = 1$, $y(1) = 2$.

4. (8pts) The differential equation $y' = 2xy^2$ is given.
a) Use Euler's method with step size 1 to estimate by hand $y(3)$, if $y(1) = -1$.
b) Use Euler's method with 10 and 50 subdivisions, respectively, to estimate $y(3)$ again. Use the exact solution $y(x) = -\frac{1}{x^2}$ to find the error of your estimates. Comment how much error dropped when you increased the number of subdivisions.

5. (10pts) Find the general solution for the differential equations:

a) $y' = \dfrac{e^{2x}}{4y^3}$

b) $y' = \cos^2 y \cos x$

6. (7pts) The half-life of radium-226 is approximately 1600 years. Suppose we start with a sample of 60mg of radium-226.
a) Write the differential equation that models the mass and show how you solve it.
b) Find the formula for the mass of the sample after t years.

7. (10pts) The "Give a Hoot" wildlife refuge is a large forest whose initial population of 40 owls grew to 100 owls in 2 years. Assume that the owl population grows logistically and that the forest can support at most 1000 owls.
a) Write the differential equation that models the owl population and write its most general solution.
b) Find the formula for the number of owls after t years. (You don't have to solve the differential equation.)
c) How long will it take for the owl population to reach 600 at "Give a Hoot"?

Bonus. (5pts) Find the orthogonal trajectory to the family of curves $y = \frac{1}{x+k}$. Sketch both collections of curves in the same coordinate system to see that they are indeed orthogonal.