Evaluate the following integrals:

1. (5pts) $\int x^2 e^x \, dx =$

2. (6pts) $\displaystyle\int \frac{x+3}{(x-1)(x+2)} \, dx =$

3. (5pts) $\displaystyle\int \sin^3 x \cos^2 x \, dx =$

4. (7pts) $\int \frac{dx}{x^2\sqrt{x^2-1}}\,dx =$

5. (7pts) Evaluate the integral $\int \frac{\ln x}{x^2}\,dx$ and use this to determine whether $\int_1^\infty \frac{\ln x}{x^2}\,dx$ converges.

6. (5pts) Find the surface area of the surface obtained by rotating about the x-axis the segment joining points $(1, 3)$ and $(3, 7)$.

7. (10pts) The integral $\int_0^3 e^{2x}\,dx$ is given.
a) Compute the exact value of the integral.
b) Use the error estimate $|E_S| \leq \frac{K(b-a)^5}{180n^4}$ to estimate the error that Simpson's rule makes for $n = 20$.
c) Compute the exact error for S_{20} and verify that it compares correctly to b).
d) What should n be in order for S_n to give you an error less than 10^{-6}?

8. (5pts) Use comparison to find out whether $\displaystyle\int_1^\infty \frac{1+\sin^2 x}{x}\,dx$ converges.

Note: only the bonus problem with the higher score will count toward your test total.

Bonus 1. (5pts) Evaluate $\displaystyle\int \frac{\sqrt{x}}{\sqrt{x}+2}\,dx =$

Bonus 2. (5pts) Simpson's rule for 2 subintervals is $S_2 = \frac{\Delta x}{3}(y_0 + 4y_1 + y_2)$. Show algebraically that this will always give an exact answer for $\int_a^b x^3\,dx$. Hint: recall that y_0, y_1 and y_2 are, respectively, values of the function at the left endpoint, the midpoint and the right endpoint of the interval $[a, b]$.