Spring '03/MAT 308/Exam 1 **Name:** *Show all your work.*

1. (7pts) Sketch the region and find its area if it is enclosed by the curves $y = x^2 + 1$, $y = 3 - x^2$.

2. (7pts) Sketch the region and find its area if it is enclosed by the curves $y = \frac{1}{x}$, $y = 1$, $y = 3$ and $y = -x$.

3. (8pts) Sketch the region enclosed by the curves $y = x^2$ and $y = 3x$. Find the volume of the solid obtained by rotating this region about the x-axis. Sketch the solid and a typical cross-section or cylindrical shell, depending on the method you are using.

4. (6pts) The base of a solid is the region bounded by the lines $x = 3$, $y = \frac{1}{3}x + 1$ and the x and y axes. Set up the integral for the volume of the solid if its cross-sections perpendicular to the x-axis are half-disks. Sketch the solid and a typical cross-section.

5. (6pts) Sketch the region enclosed by the curves $y = 2^x$, $y = 3^x$, $x = 1$ and $x = 2$. Set up the integral for the volume of the solid obtained by rotating this region about the y-axis. Sketch the solid and a typical cross-section or cylindrical shell, depending on the method you are using.

6. (8pts) A cable that weighs 2lb/ft is used to lift 800lb of coal up a mine shaft 500ft deep. Find the work done.

7. (8pts) Consider the function $f(x) = \sin x$ over the interval $[0, \pi]$.
a) What is the average value of the function over the interval?
b) What is the geometric interpretation of average value? Sketch a picture and verify that the picture is plausible in light of the geometric interpretation.
c) Verify the conclusion of the Mean Value Theorem for integrals, that is, find values of c in the interval so that $f_{\text{ave}} = f(c)$ (you will need to use the calculator).

Bonus (5pts) Set up the integral for the volume of a solid torus. It has the shape of a doughnut and is obtained by rotating a disk around an axis (see picture). Assume the disk rotated has radius b and its center is distance a from the axis of rotation.