1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x\to 1^+} f(x) =$

$\lim_{x\to 3^-} f(x) =$

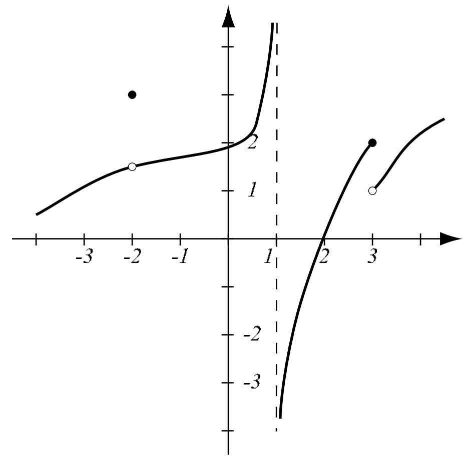
$\lim_{x\to 3^+} f(x) =$

$\lim_{x\to 3} f(x) =$

$\lim_{x\to -2} f(x) =$

$f(-2) =$

List points where f is not continuous and justify why it is not continuous at those points.



2. (4pts) Find the following limit algebraically (no need to justify, other than showing the computation).

$\lim_{x\to 2} (x^2 + 4x - 6) =$

3. (8pts) Let $\lim_{x\to 3} f(x) = 4$ and $\lim_{x\to 3} g(x) = 7$. Use limit laws to find the limit below and show each step.

$$\lim_{x\to 3} \frac{3f(x) + 7}{x^2 - f(x)g(x)} =$$

4. (10pts) Find the domain of $f(x) = \dfrac{x^2}{\sqrt{e^x}}$. Then explain, using continuity laws, why the function is continuous on its domain.

5. (16pts) The height of a turnip t seconds after getting thrown upwards with initial velocity 20 meters per second is given by $h(t) = 20t - 5t^2$ (in meters).
a) Find the average velocities of the object over six short intervals of time, three of them beginning with 1.5, and three ending with 1.5. Show the table of values.
b) Use the information in a) to find the instantaneous velocity of the turnip at $t = 1.5$.

6. (10pts) Find the following limit algebraically (do not use the calculator) and justify.

$$\lim_{x \to 4-} \frac{5x+3}{x-4} =$$

7. (18pts) The equation $e^x = x^2$ is given.
a) Use the Intermediate Value Theorem to show that this equation has at least one solution. Write a nice sentence that shows how you are using the IVT.
b) Use the bisection method and your calculator to find an interval of width less than 0.05 that contains your solution. Show every intermediate step.

8. (8pts) Let $f(x) = x^2 - 3x + 5$, and let $P = (4, 9)$. If $Q = (x, f(x))$ is another (general) point on the graph of f, write the formula for the slope of the secant line PQ and simplify.

9. (10pts) Consider the limit $\lim_{x \to 0} \frac{\sin x - x}{x^3}$.

a) Use your calculator to find the limit correct to six decimal places — write down the table on paper. Make a guess as to what the limit is exactly.

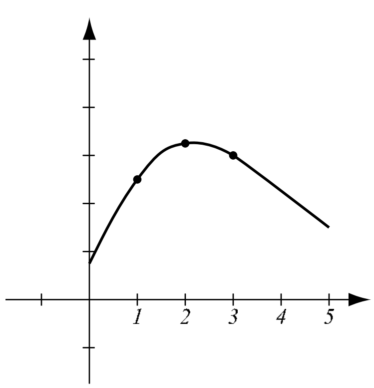
b) What does the calculator give you if you take an x very close to 0? Does this alter your estimate of the limit? Why or why not?

Bonus. (10pts) Below is the graph of the outdoor temperature (in °C) measured t hours after noon. Use it to put the following three numbers in increasing order. No computation is needed, but justify your answer.

$a =$ average rate of change from $t = 1$ to $t = 2$

$b =$ average rate of change from $t = 1$ to $t = 3$

$c =$ instantaneous rate of change at $t = 1$



Spring '10/MAT 250/Exam 2 **Name:** *Show all your work.*

1. (16pts) Differentiate and simplify where appropriate:

$$\frac{d}{dx}\left(7x^4 - \frac{1}{x^8} + \sqrt[5]{x^{17}} + \sqrt{7}\right) =$$

$$\frac{d}{dx}\,\frac{4x^2 + 5\sqrt[3]{x} + 1}{\sqrt{x}} =$$

$$\frac{d}{dt}\left(Ae^t + B\right) =$$

2. (14pts) Find the equation of the tangent line to the curve $y = x^2 - 2x - 8$ at the point $a = 0$. Sketch the curve and the tangent line.

3. (22pts) Find the following limits algebraically.

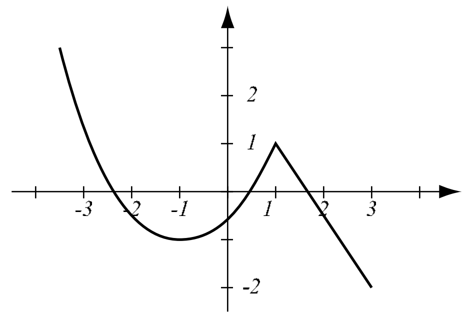
$$\lim_{x\to 7} \frac{x^2 - 2x - 35}{x - 7} =$$

$$\lim_{x\to 9} \frac{x - 9}{\sqrt{x} - 3} =$$

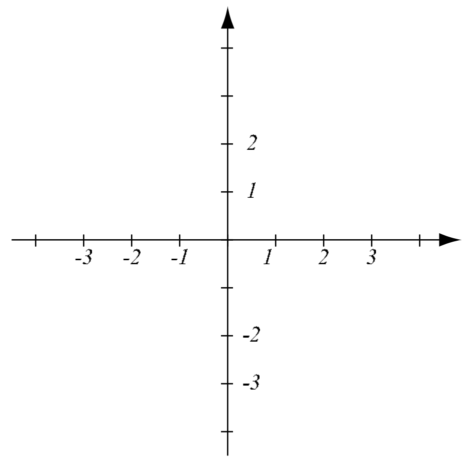
$$\lim_{x\to 0} \frac{\tan(5x)}{\sin(2x)} =$$

4. (10pts) Find $\lim\limits_{x\to 0+} (e^x - 1)\left(1 + \cos\frac{1}{x}\right)$. Use the theorem that rhymes with what is halfway between your feet and hips.

5. (16pts) The graph of the function $f(x)$ is shown at right.



a) Use the graph to fill out the table below with your estimates, noting if any of the derivatives do not exist.
b) In the second coordinate system, sketch the graph of the function $f'(x)$, with help from results in a).



x	-2	-1	0	1	2
$f'(x)$					

6. (12pts) Let $f(x) = 5x^2 + 2x - 1$.
a) Use the limit definition of the derivative to find the derivative of the function.
b) Check your answer by taking the derivative of f.

7. (10pts) Consider the limit below. It represents a derivative $f'(a)$.
a) Find f and a.
b) Use the infomation above to find the limit.

$$\lim_{x\to 2} \frac{x^3 - 8}{x - 2}$$

Bonus. (10pts) Let $m_b = \lim_{x\to 0} \frac{b^h - 1}{h}$ and let $f(x) = b^x$, $b > 0$, $b \neq 1$.
a) Use the limit definition of the derivative to show that $f'(0) = m_b$.
b) Use the limit definition of the derivative to show that $f'(x) = m_b \cdot b^x$.

Spring '10/MAT 250/Exam 3 **Name:** *Show all your work.*

Differentiate and simplify where appropriate:

1. (7pts) $\dfrac{d}{dx}\left((x^3 + 4x^2)e^x\right) =$

2. (8pts) $\dfrac{d}{dx}\dfrac{x^2 + 5x - 4}{3x - 5} =$

3. (7pts) $\dfrac{d}{dt}\dfrac{1}{\sqrt{t^4 + 4t^2 + 1}} =$

4. (8pts) $\dfrac{d}{d\theta}\dfrac{e^{-2\theta}}{\sin(5\theta)} =$

5. (8pts) $\dfrac{d}{dx}\tan\left(\sqrt{x^7 - \sqrt[3]{x}}\right) =$

6. (10pts) Find the equation of the tangent line to the curve $y = \sin^2 x$ at the point $x = \frac{\pi}{3}$.

7. (14pts) A golf ball is shot upward with inital velocity 30 meters per second.
a) Write the formula for the position of the ball at time t (you may assume $g \approx 10$).
b) Write the formula for the velocity of the ball at time t.
c) When does the ball have upward velocity $10\frac{m}{s}$?. At what height is it at that moment?

8. (14pts) The body surface area A (in meters squared) of a person 196cm tall is given by the formula $A = \frac{7\sqrt{w}}{30}$, where w is the weight of the person in kilograms.
a) Find the body surface area of a person weighing 81kg.
b) Find the ROC of the body surface area with respect to weight when $w = 81$ (units?).
c) Use b) estimate the change in body surface area if weight changes by 2kg.
d) Use c) to estimate the body surface area of a person weighing 83kg .

9. (12pts) Find $g''\left(\frac{\pi}{4}\right)$, if $g(x) = \dfrac{\sin x}{\sin x + \cos x}$.

10. (12pts) Let $f(x) = x^{-\frac{3}{2}}$.
a) Find the first four derivatives of f.
b) Find the general formula for $f^{(n)}(x)$.

Bonus. (10pts) Use the product rule to establish the quotient rule. To do this, let $h = \frac{f}{g}$, so $hg = f$. Take the derivative of both sides of the last equation, and solve for h', expressing the solution only in terms of f and g.

Spring '10/MAT 250/Exam 4 **Name:** *Show all your work.*

Differentiate and simplify where appropriate:

1. (5pts) $\dfrac{d}{dx}\ln(x^2 - 3x + 1) =$

2. (5pts) $\dfrac{d}{d\theta}5^{\sin\theta} =$

3. (6pts) $\dfrac{d}{dx}x^2 \arcsin x =$

4. (9pts) $\dfrac{d}{dx}\ln\dfrac{(x+1)^2}{(3x-2)^4} =$

5. (9pts) $\dfrac{d}{dt}\arctan\left(\dfrac{t^2-1}{t}\right) =$

6. (12pts) Use implicit differentiation to find the equation of the tangent line to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ at the point $(-2, \frac{3\sqrt{3}}{2})$. Draw the picture of the ellipse and the tangent line.

7. (10pts) Use implicit differentiation to find y'.

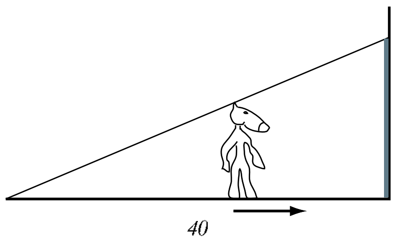
$$xe^y = \frac{x}{y}$$

8. (8pts) Use logarithmic differentiation to find the derivative of $y = x^{\cos x}$.

9. (10pts) Let $f(x) = x^3 + 2x^2 + 5x$, and let g be the inverse of f. Use the theorem on derivatives of inverses to find $g'(8)$.

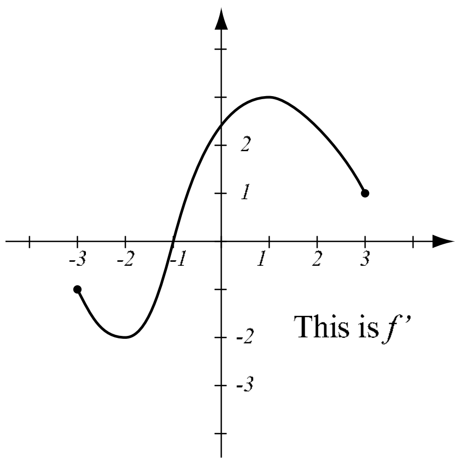
10. (10pts) Find all points on the graph of $3x^2 + 4y^2 + 3xy = 24$ where the tangent line is horizontal.

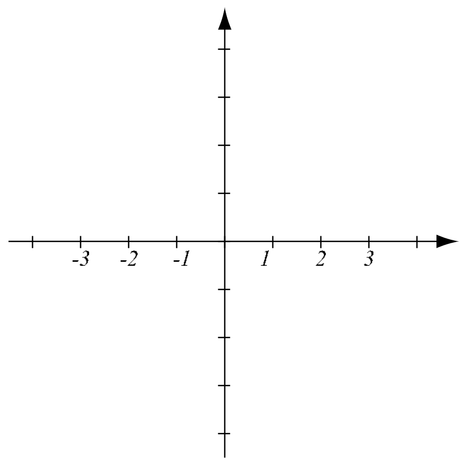
11. (16pts) Somewhere on campus, a spotlight is shining on a wall 40ft away. Seven-foot tall Dunker is walking from the spotlight toward the wall at rate 4 ft/second. How fast is the height of Dunker's shadow on the wall changing when Dunker is 30ft away from the spotlight? What are the units?



Bonus. (10pts) Let $f(\theta) = \sec\theta$, and let g be the inverse of f. Use the theorem on derivatives of inverses to find the general expression for $g'(x)$. (A triangle with θ and x will help you.)

1. (19pts) Let the domain of f be $[-3,3]$. The graph of its derivative f' is drawn below. Use the graph to answer:
a) What are the intervals of increase and decrease of f? Where does f have a local minimum or maximum?
b) What are the intervals of concavity of f? Where does f have inflection points?
c) Use the information gathered in a) and b) to draw one possible graph of f at right.





2. (10pts) A rubber ball is being inflated. Using linear approximation, estimate by how much volume changes if radius changes from $r = 6$in to $r = 6.5$in. (The volume of a ball of radius r is $V = \frac{4\pi}{3}r^3$.)

3. (13pts) Verify the Mean Value Theorem for the function $f(x) = \sqrt{x}$ on the interval $[1, 4]$.

4. (21pts) Let $f(x) = \dfrac{x+3}{x^2+7}$.

a) Find the cricital points and the intervals of increase and decrease of f. Determine where f has a local maximum or minimum.

b) Using information found in a), draw a rough sketch of f.

c) _____ Initial here if you are glad you were not asked to find the second derivative of f. (But if you really want to, see the bonus).

5. (16pts) Let $f(\theta) = \sin^2\theta - \cos\theta$. Find the absolute minimum and maximum values of f on the interval $[0, \pi]$.

6. (21pts) Let $f(x) = e^{-x}(2x + 3)$.
a) Find the intervals of concavity and points of inflection for f.
b) Find the critical points and use a) to determine which are local minima or maxima.

The better of these two bonus problems will count toward your grade.

Bonus. (8pts) After two hours of driving, Frank was 70 miles away from his starting point. After three hours he was 120 miles from the starting point. There are three values that you can be sure his speedometer displayed during his drive. What are they? Justify your answer with a theorem. (Don't say one of the values is zero, because he might not have been at a standstill when we started observing.)

Bonus. (10pts) Find the intervals of concavity for the function in problem 4.

Spring '10/MAT 250/Exam 6 **Name:** *Show all your work.*

Find the limits. You may use L'Hopital's rule.

1. (10pts) $\lim_{x \to \infty} \dfrac{x^2 + 3x - 7}{e^x} =$

Which grows faster, $x^2 + 3x - 7$ or e^x?

2. (8pts) $\lim_{x \to 0} \dfrac{\cos x - 1}{x^2} =$

3. (10pts) $\lim_{x \to 0+} \sqrt[5]{x} \ln x =$

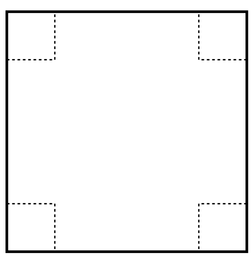
4. (10pts) $\lim_{x \to \infty} x^{\frac{1}{x}} =$

5. (32pts) Let $f(x) = \ln(x^2 + 9)$. Draw an accurate graph of f by following the guidelines.
a) Find the intervals of increase and decrease, and local extremes.
b) Find the intervals of concavity and points of inflection.
c) Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
d) Use information from a)–d) to sketch the graph

6. (8pts) Find the limit without using L'Hopital's rule.

$$\lim_{x\to\infty} \frac{5x^2 - 3x + 1}{2x^2 - x + 2} =$$

7. (22pts) A square piece of cardboard has side length 4ft. Four equal-size square pieces are cut from the corners to produce a cross-like shape, whose "flaps" are lifted up to form a box without a top. Find the size of the cutout that produces the box with the largest volume.



Bonus. (10pts) Draw a line of negative slope through the point $(2, 3)$. Along with the axes, this line forms a right triangle in the first quadrant. Among all triangles obtained in this way, find the one with the smallest area.

Spring '10/MAT 250/Exam 7 **Name:** *Show all your work.*

Find the following antiderivatives.

1. (4pts) $\int 4x^2 - 3\sqrt{x}\, dx =$

2. (4pts) $\int e^{3x+1}\, dx =$

3. (4pts) $\int \sec(4x)\tan(4x)\, dx =$

4. (15pts) Find $\int_{-2}^{2} x - 1\, dx$ in two ways (they'd better give you the same answer!):
a) Using the "area" interpretation of the integral. Draw a picture.
b) Using the Fundamental Theorem of Calculus.

5. (6pts) Write in sigma notation.

$$\frac{2}{3} + \frac{4}{9} + \frac{6}{27} + \frac{8}{81} + \frac{10}{243} =$$

6. (10pts) Evaluate the definite integral:

$$\int_1^4 \frac{x^2 - 1}{\sqrt{x}}\, dx =$$

Use the substitution rule in the following integrals:

7. (9pts) $\int (6x^2 - 4x)(x^3 - x^2 + 2)^5 \, dx =$

8. (10pts) $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sec^2 x}{\tan^7 x}\, dx =$

9. (8pts) Find the function f if $f'(x) = \frac{5}{x^4} - \frac{3}{x}$, and $f(1) = 7$.

10. (22pts) The function $f(x) = \cos x$, $0 \leq x \leq \frac{\pi}{2}$ is given.
a) Write down the expression that is used to compute R_4. Then compute R_4.
b) Illustrate with a diagram, where appropriate rectangles are clearly visible. What does R_4 represent?
c) Is R_4 an overestimate or an underestimate of the area?
d) Using the Fundamental Theorem of Calculus, evaluate $\int_0^{\frac{\pi}{2}} \cos x \, dx$. What is the error of R_4?

11. (8pts) Show that $0.82 \le \int_1^{1.3} e^{x^2}\,dx \le 1.63$. (Note: the antiderivative of e^{x^2} cannot be found among elementary functions, so don't try to do it by evaluating the integral.)

Bonus. (10pts) The gist of section 5.5 is this:
$\int_a^b$ rate of change of F = change of F from a to b. In other words, $\int_a^b F'(x)\,dx = F(b) - F(a)$.

Use the fact above to solve the following problem. Water flows in and out of a tank at rate $3 - \frac{1}{2}t$ liters/minute. There were 5 liters of water in the tank at time $t = 0$.
a) By how much does the amount of water in tank change from $t = 0$ to $t = 4$?
b) How much water is in the tank at time $t = 4$?
c) By how much does the amount of water in tank change from $t = 4$ to $t = 10$?
d) How much water is in the tank at time $t = 10$?

Find the following limits algebraically, without using L'Hopital's rule.

12. (7pts) $\lim\limits_{x \to 16} \dfrac{\sqrt{x} - 4}{x - 16} =$

13. (7pts) $\lim\limits_{x \to \infty} \dfrac{3x^2 + 2x + 17}{x^3 - 5x^2 + 7x - 1} =$

14. (8pts) Find the limit. You may use L'Hopital's rule.

$$\lim_{x \to 0} \frac{\tan(5x)}{\sin(2x)} =$$

Differentiate and simplify where appropriate:

15. (4pts) $\dfrac{d}{dx} \ln(\arctan x) =$

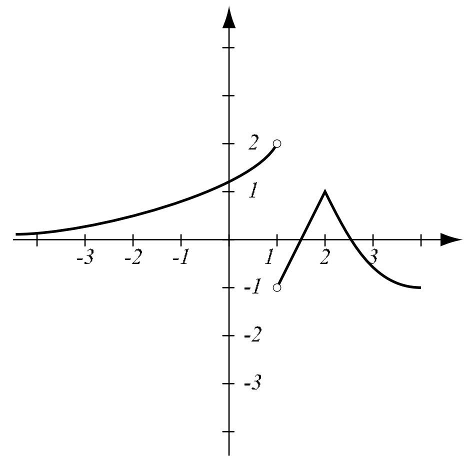
16. (5pts) $\dfrac{d}{d\theta} 7^{\sec \theta} =$

17. (11pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x\to 1^-} f(x) =$

$\lim_{x\to 1^+} f(x) =$

$\lim_{x\to 1} f(x) =$



List points where f is not continuous (justify).

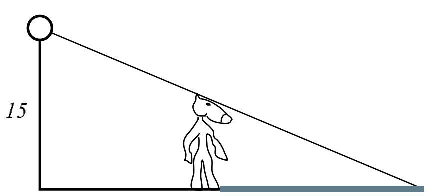
List points where f is not differentiable (justify).

18. (16pts) The height of Rogawski's book, t seconds after getting celebratorily thrown upwards with initial velocity 30 meters per second, is given by $h(t) = 30t - 5t^2$ (in meters).
a) Write the expressions for average velocities of the book over two short intervals of time, both beginning with $t = 2$. (Do not evaluate).
b) Use a limit process to find the instantaneous velocity of the flying Rogawski at $t = 2$. Is it going upwards or downwards?

19. (32pts) Let $f(x) = e^{-x}(2x+5)$. Draw an accurate graph of f by following the guidelines.
a) Find the intervals of increase and decrease, and local extremes.
b) Find the intervals of concavity and points of inflection.
c) Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$. (Use L'Hopital if necessary.)
d) Use information from a)–c) to sketch the graph. (Leave significant y-coordinates in closed form — don't evaluate.)

20. (12pts) The body surface area A (in meters squared) of a person 169cm tall is given by the formula $A = \frac{13\sqrt{w}}{60}$, where w is the weight of the person in kilograms.
a) Find the body surface area of a person weighing 64kg.
b) Find the ROC of the body surface area with respect to weight when $w = 64$ (units?).
c) Use b) estimate the change in body surface area if weight changes by 3kg.
d) Use c) to estimate the body surface area of a person weighing 67kg .

21. (14pts) Somewhere on campus, seven-foot tall Dunker is walking away from a lamppost 15ft tall at a speed of 4 ft/second. How fast is length of Dunker's shadow on the ground changing when Dunker is 30ft away from the lamppost? What are the units?



22. (12pts) Use implicit differentiation to find the equation of the tangent line to the curve $x^2 + 3\frac{y}{x} + y^2 = 11$ at the point $(1, 2)$.

23. (12pts) The integral $\int_1^3 x^2 - 2x\,dx$ is given.
a) Draw a graph of $y = x^2 - 2x$ and state what the integral represents graphically. Can you tell from the picture if the integral is positive or negative?
b) Evaluate the integral and confirm your answer from a).

24. (10pts) Use the substitution rule to find the integral:

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin x}{\cos^3 x}\, dx =$$

Bonus. (15pts) Draw a line of negative slope through the point $(2, 3)$. Along with the axes, this line forms a right triangle in the first quadrant. Among all triangles obtained in this way, find the one with the smallest area.