Spring '10/MAT 250/Exam 3 **Name:** Solution *Show all your work.*

Differentiate and simplify where appropriate:

1. (7pts) $\frac{d}{dx}\left((x^3+4x^2)e^x\right) = (3x^2+8x)e^x + (x^3+4x^2)e^x$

$= e^x(x^3+7x^2+8x)$

2. (8pts) $\frac{d}{dx}\frac{x^2+5x-4}{3x-5} = \frac{(2x+5)(3x-5)-(x^2+5x-4)\cdot 3}{(3x-5)^2} = \frac{6x^2+5x-25-(3x^2+15x-12)}{(3x-5)^2}$

$= \frac{3x^2-10x-13}{(3x-5)^2}$

3. (7pts) $\frac{d}{dt}\frac{1}{\sqrt{t^4+4t^2+1}} = \frac{d}{dt}(t^4+4t^2+1)^{-\frac{1}{2}} = -\frac{1}{2}(t^4+4t^2+1)^{-\frac{3}{2}}\cdot(4t^3+8t)$

$= -\frac{4(t^3+2t)}{2(t^4+4t^2+1)^{3/2}} = -\frac{2(t^3+2t)}{(t^4+4t^2+1)^{3/2}}$

4. (8pts) $\frac{d}{d\theta}\frac{e^{-2\theta}}{\sin(5\theta)} = \frac{e^{-2\theta}(-2)\sin(5\theta)-e^{-2\theta}\cos(5\theta)\cdot 5}{\sin^2(5\theta)} = -\frac{e^{-2\theta}(2\sin(5\theta)+5\cos(5\theta))}{\sin^2(5\theta)}$

5. (8pts) $\frac{d}{dx}\tan\left(\sqrt{x^7-\sqrt[3]{x}}\right) = \sec^2\left(\sqrt{x^7-\sqrt[3]{x}}\right)\cdot\frac{1}{2\sqrt{x^7-\sqrt[3]{x}}}\cdot\left(7x^6-\frac{1}{3}x^{-\frac{2}{3}}\right)$

$\uparrow$ $x^{\frac{1}{3}}$

$= \frac{\left(7x^6-\frac{1}{3}x^{-\frac{2}{3}}\right)\sec^2\sqrt{x^7-\sqrt[3]{x}}}{\sqrt{x^7-\sqrt[3]{x}}}$

6. (10pts) Find the equation of the tangent line to the curve $y = \sin^2 x$ at the point $x = \frac{\pi}{3}$.

$y' = 2\sin x \cos x$

$y'\left(\frac{\pi}{3}\right) = 2 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2} = \frac{\sqrt{3}}{2}$

$y\left(\frac{\pi}{3}\right) = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$

Equation of tangent line:

$y - \frac{3}{4} = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right)$

$y = \frac{\sqrt{3}}{2}x - \frac{\pi\sqrt{3}}{6} + \frac{3}{4}$

7. (14pts) A golf ball is shot upward with inital velocity 30 meters per second.
a) Write the formula for the position of the ball at time t (you may assume $g \approx 10$).
b) Write the formula for the velocity of the ball at time t.
c) When does the ball have upward velocity $10\frac{m}{s}$?. At what height is it at that moment?

a) $s(t) = 30t - \frac{1}{2} \cdot 10t^2$

$= 30t - 5t^2$

b) $v(t) = s'(t) = 30 - 10t$

c) $30 - 10t = 10$

$20 = 10t$

$t = 2s$

$s(2) = 30 \cdot 2 - 5 \cdot 2^2$

$= 60 - 20 = 40m$

8. (14pts) The body surface area A (in meters squared) of a person 196cm tall is given by the formula $A = \frac{7\sqrt{w}}{30}$, where w is the weight of the person in kilograms.
a) Find the body surface area of a person weighing 81kg.
b) Find the ROC of the body surface area with respect to weight when $w = 81$ (units?).
c) Use b) estimate the change in body surface area if weight changes by 2kg.
d) Use c) to estimate the body surface area of a person weighing 83kg .

a) $A(81) = \frac{7\sqrt{81}}{30} = \frac{63}{30} = \frac{21}{10} = 2.1\ m^2$

b) $\frac{dA}{dw} = \frac{d}{dw}\left(\frac{7}{30}\sqrt{w}\right) = \frac{7}{30}\cdot\frac{1}{2\sqrt{w}} = \frac{7}{60\sqrt{w}}$

$A'(81) = \frac{7}{60\cdot\sqrt{81}} = \frac{7}{540}\ m^2/kg$

c) $\Delta A \approx A'(81)\cdot\Delta w = \frac{7}{540}\cdot 2 = \frac{7}{270}\ m^2$

d) $A(83) \approx A(81) + \frac{7}{270} = \frac{21}{10} + \frac{7}{270} = \frac{567+7}{270} = \frac{574}{270} = \frac{287}{135}\ m^2$

$27\cdot 21$
54
27
567

9. (12pts) Find $g''\left(\frac{\pi}{4}\right)$, if $g(x) = \dfrac{\sin x}{\sin x + \cos x}$.

$g'(x) = \frac{\cos x(\sin x + \cos x) - \sin x(\cos x - \sin x)}{(\sin x + \cos x)^2} = \frac{\cos^2 x + \sin^2 x}{(\quad)^2} = \frac{1}{(\sin x + \cos x)^2} = (\sin x + \cos x)^{-2}$

$g''(x) = -2(\sin x + \cos x)^{-3}\cdot(\cos x - \sin x) = \frac{2(\sin x - \cos x)}{(\sin x + \cos x)^3}$

$g''\left(\frac{\pi}{4}\right) = \frac{2\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)^2} = 0$

10. (12pts) Let $f(x) = x^{-\frac{3}{2}}$.
a) Find the first four derivatives of f.
b) Find the general formula for $f^{(n)}(x)$.

a) $y = x^{-\frac{3}{2}}$

$y' = -\frac{3}{2} x^{-\frac{5}{2}}$

$y'' = \left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right) x^{-\frac{7}{2}}$

$y''' = \left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(-\frac{7}{2}\right) x^{-\frac{9}{2}}$

2 less than exponent

$y'''' = \left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(-\frac{7}{2}\right)\left(-\frac{9}{2}\right) x^{-\frac{11}{2}}$

b) $y^{(n)} = (-1)^n \dfrac{3 \cdot 5 \cdot 7 \cdot \ldots \cdot (2n+1)}{2^n} x^{-\frac{2n+3}{2}}$

n	number
1	5
2	7
3	9
4	11
	2n+3

Bonus. (10pts) Use the product rule to establish the quotient rule. To do this, let $h = \frac{f}{g}$, so $hg = f$. Take the derivative of both sides of the last equation, and solve for h', expressing the solution only in terms of f and g.

$hg = f \quad \Big| \frac{d}{dx}$

$h'g + hg' = f'$

$h'g = f' - hg' \quad \Big| \div g$

$h' = \dfrac{f' - hg'}{g} = \dfrac{f' - \frac{f}{g} g'}{g} \cdot \dfrac{g}{g} = \dfrac{f'g - fg'}{g^2}$