1. (12pts) Simplify and write the answer so all exponents are positive:
a) $\left(x^{4} y^{7}\right)^{-2} x^{11} y^{5}=$
b) $\frac{5 x^{3}(2 y)^{4}}{\left(10 x^{2} y^{-4}\right)^{3}}=$
2. (4pts) Convert to scientific notation or a decimal number:
$1.4375 \times 10^{8}=$
$0.000000326=$
3. (8pts) Simplify and write in standard form:
a) $(3 x-1)(2 x+5)-x(7 x+4)=$
b) $(x+2)\left(3 x^{2}-4 x+5\right)=$
4. (15pts) Use formulas to expand:
a) $(2 x-5)(2 x+5)=$
b) $(3 x-8)^{2}=$
c) $(4 x+3)^{3}=$
5. (14pts) Factor the following. Use either a known formula or a factoring method.
a) $x^{2}-4 x-32=$
b) $18 x^{2}-9 x-5=$
c) $x^{3}+125=$
6. (7pts) Find the two errors in the computation. Then redo the computation correctly. $(x+7)^{2}-3(x-5)=x^{2}+49-3 x-15=x^{2}-3 x+34$
7. (16pts) Simplify.
$\frac{x-1}{x^{2}+2 x-15}-\frac{2 x-1}{3 x^{2}+14 x-5}=$
$\frac{3+\frac{3}{x-4}}{x^{2}+4 x+3} \cdot \frac{x^{2}-16}{x-3}=$
8. (9pts) Simplify, showing intermediate steps.
$\sqrt{72}=$
$81^{\frac{3}{4}}=$
$(-32)^{\frac{2}{5}}$
9. (14pts) Simplify. Express answers in terms of positive exponents.
$\sqrt[4]{48 x^{16} y^{10}}=$
$\frac{x^{\frac{3}{4}}\left(25 x^{\frac{5}{4}} y^{-\frac{2}{3}}\right)^{\frac{3}{2}}}{10\left(36 y^{\frac{8}{3}}\right)^{-\frac{1}{2}}}=$
10. (6pts) Rationalize the denominator.
$\frac{3 \sqrt{7}-1}{5+\sqrt{7}}=$
11. (10pts) Simplify.
$(4+i)(3-5 i)=$
$\frac{3-4 i}{4+3 i}=$
12. (5pts) Simplify and justify your answer.
$i^{1002}=$
13. (18pts) Solve the equations.
$12-5(5-x)=3(x+2)-11$

$$
\frac{x-3}{3}+\frac{3 x+5}{4}=2+\frac{3-x}{6}
$$

$\frac{5 x-11}{x-6}+3=\frac{9}{x-6}$
2. (14pts) You inherit $\$ 13,000$ and decide to invest the money in two different investments, one paying $10 \%$, and the other paying $14 \%$. After a six months, your investments are worth $\$ 13,741$. How much did you invest in each account?
3. (14pts) How many liters of a $10 \%$ solution of hydrochloric acid must be mixed with 3 liters of a $24 \%$ solution of HCl in order to get a $14 \%$ solution?
4. (14pts) Fred, who is from Seattle, went to school in Florida. On the way to school, he took a southern route, and on his return after graduation, he took a northern route. On both trips he averaged the same speed. If the southern trek took 52 hours, the northern 60 hours, and the northern trek was 448 miles longer, how long was each trip?

1. (23pts) Solve the equations.
$3 x^{2}-x=5 x+11$

$$
x-2 \sqrt{11-x}=3
$$

$x^{6}+3 x^{3}=40$
2. ( 6 pts ) Solve by completing the square.
$x^{2}+16 x=19$
3. (4pts) Solve the equation.
$|4 x+1|=7$
4. (12pts) Solve the inequalities. Draw your solution and write it in interval form.
$5 \leq 3-2 x<15$

$$
|3 x+2| \geq 4
$$

5. (15pts) A landscaper has 72 cubic feet of stones that he will use to fill two circular areas to a depth of $\frac{1}{2}$ foot. If the bigger circular area has a radius that is 1 foot larger than the radius of the smaller one, what are the radii of the areas?
6. (18pts) Let $A=(2,0), B=(-1, \sqrt{3}), C=(-1,-\sqrt{3})$.
a) Draw the picture.
b) Show algebraically that the triangle $A B C$ is equilateral (all sides have equal length).
c) Use the Pythagorean theorem to show that the triangle $A P C$ is right, where $P$ is the midpoint of $A B$.
7. (12pts) Use your calculator to accurately sketch the graph of $y=x^{4}-7 x^{2}+3 x+7$. Draw the graph here, and indicate the viewing window. Find all the $x-$ and $y$-intercepts (accuracy: 4 decimal points).
8. (8pts) Find the equation of the line (in form $y=m x+b$ ) that passes through points $(-1,-2)$ and $(2,3)$.
9. (14pts) Find the equation of the line (in form $y=m x+b$ ) that is parallel to the line $2 x-3 y=7$, and passes through point $(-1,3)$. Draw both lines.
10. (8pts) Show that the triangle $A P C$ from problem 1 is right using another method, by showing that lines $A B$ and $P C$ are perpendicular (show that the product of their slopes is -1 ).
11. (14pts) Use the graph of the function $f$ at right to answer the following questions.
a) What is the domain of $f$ ?
b) What is the range of $f$ ?
c) Find $f(3)$ and $f(-1)$.
d) What are the solutions of the equation $f(x)=4$ ?
e) Find intervals where $f(x)<0$.

12. (11pts) Suppose $(-1,2)$ and $(5,-4)$ are endpoints of a diameter of a circle. Find the equation of the circle and draw the circle.
13. (10pts) Find the domain of $f(x)=\frac{\sqrt{3 x+2}}{x^{2}-3 x-40}$. Write your answer in interval notation.
14. (9pts) Sketch the graph of the piecewise-defined function:
$f(x)= \begin{cases}\frac{3}{2} x+2, & \text { if }-4 \leq x \leq 1 \\ -x+4, & \text { if } 1<x .\end{cases}$
15. (16pts) Let $f(x)=-x^{3}+13 x$ (answer with 4 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of $f$ (on paper!). Indicate scale on the graph.
b) Determine algebraically whether $f$ is even, odd, or neither. Justify your answer further by examining the graph.
c) Find where $f$ has a local minimum and maximum.
d) Find the intervals of increase and decrease.

## Spring '10/MAT 140/Joysheet 7 Name:

1. (15pts) The graph of $f(x)$ is drawn below. On three separate graphs, sketch the graphs of the functions $f(x-1), 2 f(x)$ and $f(-x)-3$ and label all the relevant points.

2. (18pts) Let $f(x)=\sqrt{x^{2}+x}, g(x)=3 x+2$. Find the following functions (simplify where possible):
$(f \cdot g)(x)=$

$$
\frac{f}{g}(3)=
$$

$(g \circ f)(-3)=$

$$
(f \circ g)(x)=
$$

$(g \circ g)(x)=$

$$
(f \circ f)(x)=
$$

3. (8pts) Consider the function $h(x)=7 x+5$. Find functions $f$ and $g$ so that $h(x)=$ $f(g((x))$. Find two different solutions to this problem, neither of which is the "stupid" one.
4. (7pts) The graph of a function $f$ is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of $f^{-1}$, labeling the relevant points.

5. (12pts) Let $g(x)=\frac{2 x+3}{x-5}$. Find the formula for $g^{-1}$. Find the domain and range of $g$.
6. (8pts) Evaluate without using the calculator:
$\log _{7} 343=$
$\log _{3} \frac{1}{81}=$
$\log _{16} 2=$
$\log _{c} \sqrt[5]{c^{2}}=$
7. (4pts) Use your calculator to find $\log _{7} 21$ with accuracy 4 decimal places.
Show how you obtained your number.
8. (13pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.
$\log _{9}\left(81 x^{5} \sqrt[3]{y^{7}}\right)=$
$\ln \frac{\left(x^{2}-4 x-21\right)^{4}}{\left(x^{2}-9 x+14\right)^{5}}=$
9. (13pts) Write as a single logarithm. Simplify if possible.
$\frac{1}{3} \log \left(125 x^{6}\right)+\frac{1}{2} \log \left(4 x^{6}\right)=$
$3 \log _{4}(x-7)+\log _{4}(x+5)-2 \log _{4}\left(x^{2}-2 x-35\right)=$
10. (9pts) Find the domain of $f(x)=-\log _{6}(x+3)$. Then use transformations (explain which ones you used) to draw the graph of this function. Display asymptotes, if they exist.
11. (7pts) How much should you invest in an account bearing $4 \%$, compounded quarterly, if you wish to have $\$ 3,000$ in five years?
12. (6pts) In 2003, the state of Colima, Mexico, was hit by an earthquake. If the energy released was $6.31 \times 10^{15}$ joules, what was the magnitude of the earthquake using the Richter scale?
