Spring '10/MAT 140/Joysheet 8 **Name:** *Show all your work.*

1. (8pts) Evaluate without using the calculator:

$\log_7 343 = 3$ $\quad 7^? = 343$

$\log_3 \frac{1}{81} = -4$ $\quad 3^? = \frac{1}{81}$ $\quad 3^4 = 81$

$\log_{16} 2 = \frac{1}{4}$ $\quad 16^? = 2$ $\quad \sqrt[4]{16} = 2$

$\log_c \sqrt[5]{c^2} = \frac{2}{5}$ $\quad c^? = c^{\frac{2}{5}}$

2. (4pts) Use your calculator to find $\log_7 21$ with accuracy 4 decimal places. Show how you obtained your number.

$\log_7 21 = \frac{\ln 21}{\ln 7} = 1.5646$

3. (13pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_9\left(81x^5\sqrt[3]{y^7}\right) = \log_9 81 + \log_9 x^5 + \log_9 y^{\frac{7}{3}}$

$= 2 + 5\log_9 x + \frac{7}{3}\log_9 y$

$\ln\frac{(x^2-4x-21)^4}{(x^2-9x+14)^5} = \ln(x^2-4x-21)^4 - \ln(x^2-9x+14)^5 = 4\ln(x^2-4x-21) - 5\ln(x^2-9x+14)$

$(x-7)(x+3)$ $\quad (x-7)(x-2)$

$= 4\left(\ln(x-7)+\ln(x+3)\right) - 5\left(\ln(x-7)+\ln(x-2)\right)$

$= -\ln(x-7) + 4\ln(x+3) - 5\ln(x-2)$

4. (13pts) Write as a single logarithm. Simplify if possible.

$\frac{1}{3}\log(125x^6) + \frac{1}{2}\log(4x^6) = \log(125x^6)^{\frac{1}{3}} + \log(4x^6)^{\frac{1}{2}}$

$= \log(5x^2) + \log(2x^3) = \log(5x^2 \cdot 2x^3) = \log(10x^5)$

$3\log_4(x-7) + \log_4(x+5) - 2\log_4(x^2-2x-35) =$

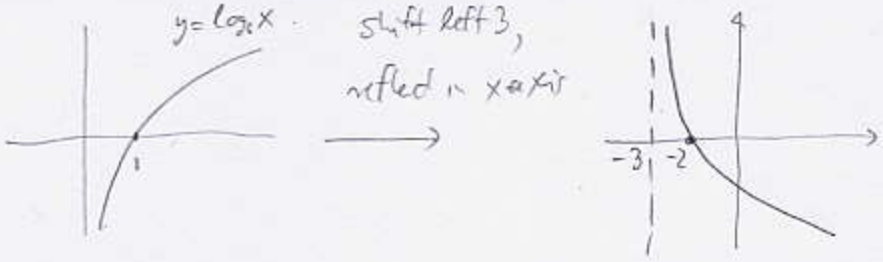
$= \log_4(x-7)^3 + \log_4(x+5) - \log_4(x^2-2x-35)^2$

$(x-7)(x+5)$

$= \log_4\frac{(x-7)^3(x+5)}{((x-7)(x+5))^2} = \log_4\frac{(x-7)^{\cancel{3}\,1}\cancel{(x+5)}}{\cancel{(x-7)^2}\,(x+5)^{\cancel{2}\,1}} = \log_4\frac{x-7}{x+5}$

5. (9pts) Find the domain of $f(x) = -\log_6(x+3)$. Then use transformations (explain which ones you used) to draw the graph of this function. Display asymptotes, if they exist.

Must have:
$x+3>0$
$x>-3$
Domain $= (-3, \infty)$



$x=-3$ is the vertical asymptote.

6. (7pts) How much should you invest in an account bearing 4%, compounded quarterly, if you wish to have \$3,000 in five years?

$$3000 = P\left(1+\frac{0.04}{4}\right)^{4\cdot 5}$$

$$3000 = P(1.01)^{20}$$

$$P = \frac{3000}{1.01^{20}} = \frac{3000}{1.2202} = 2458.63$$

7. (6pts) In 2003, the state of Colima, Mexico, was hit by an earthquake. If the energy released was 6.31×10^{15} joules, what was the magnitude of the earthquake using the Richter scale?

$$M = \frac{2}{3}\log\frac{6.31\times 10^{15}}{10^{4.4}} = \frac{2}{3}\log\left(6.31\times 10^{10.6}\right) = \frac{2}{3}\cdot 11.4$$

$$= 7.6 \text{ on Richter scale.}$$