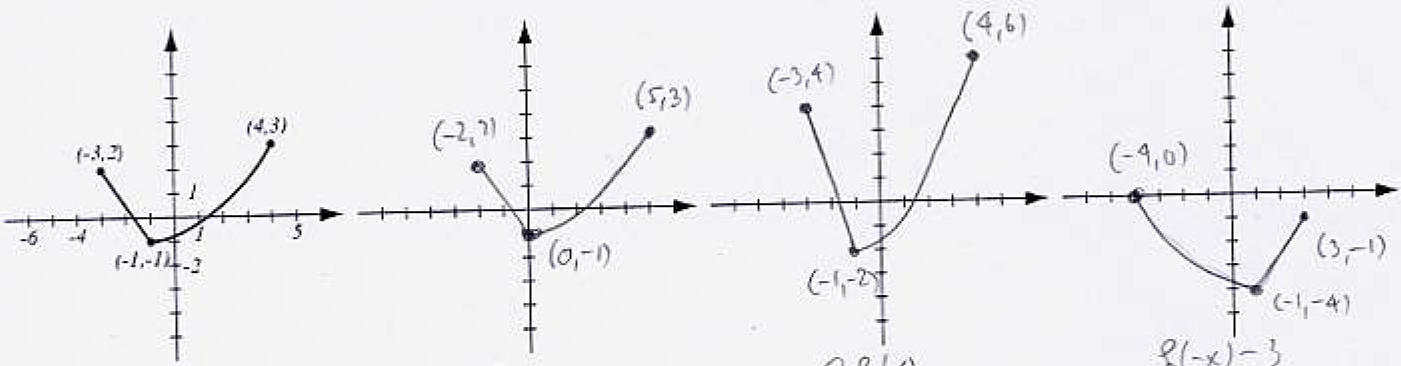
Spring '10/MAT 140/Joysheet 7 **Name:** Solution *Show all your work.*

1. (15pts) The graph of $f(x)$ is drawn below. On three separate graphs, sketch the graphs of the functions $f(x-1)$, $2f(x)$ and $f(-x)-3$ and label all the relevant points.



$f(x-1)$ shift right 1

$2f(x)$ vertical stretch, factor 2

$f(-x)-3$ reflect in y-axis, shift down 3

2. (18pts) Let $f(x)=\sqrt{x^2+x}$, $g(x)=3x+2$. Find the following functions (simplify where possible):

$$(f\cdot g)(x) = f(x)\cdot g(x) = \left(\sqrt{x^2+x}\right)\cdot(3x+2) = (3x+2)\sqrt{x^2+x}$$

$$\frac{f}{g}(3) = \frac{f(3)}{g(3)} = \frac{\sqrt{9+3}}{9+2} = \frac{\sqrt{12}}{11} = \frac{2\sqrt{3}}{11}$$

$$(g\circ f)(-3) = g(f(-3)) = g\left(\sqrt{9-3}\right) = g(\sqrt{6}) = 3\sqrt{6}+2$$

$$(f\circ g)(x) = f(g(x)) = f(3x+2) = \sqrt{(3x+2)^2+3x+2} = \sqrt{9x^2+12x+4+3x+2} = \sqrt{9x^2+15x+6}$$

$$(g\circ g)(x) = g(g(x)) = g(3x+2) = 3(3x+2)+2 = 9x+8$$

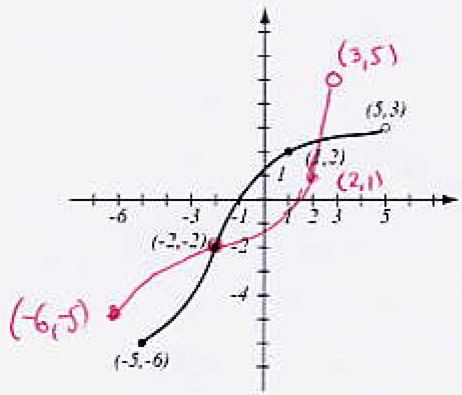
$$(f\circ f)(x) = f(f(x)) = f\left(\sqrt{x^2+x}\right) = \sqrt{\left(\sqrt{x^2+x}\right)^2+\sqrt{x^2+x}} = \sqrt{x^2+x+\sqrt{x^2+x}}$$

3. (8pts) Consider the function $h(x) = 7x + 5$. Find functions f and g so that $h(x) = f(g((x))$. Find two different solutions to this problem, neither of which is the "stupid" one.

$g(x) = 7x$ $\quad$ $g(x) = 7x + 3$ $\quad$ $g(x) = \frac{x}{2}$ $\quad$ $g(x) = x - 1$

$f(x) = x + 5$ $\quad$ $f(x) = x + 2$ $\quad$ $f(x) = 14x + 5$ $\quad$ $f(x) = 7x + 12$

4. (7pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points.



a) Yes, because it passes the horizontal line test.

b)

5. (12pts) Let $g(x) = \dfrac{2x+3}{x-5}$. Find the formula for g^{-1}. Find the domain and range of g.

$y = \dfrac{2x+3}{x-5}$ solve for x

$y(x-5) = 2x+3$

$yx - 5y = 2x + 3 \quad | -2x, +5y$

$yx - 2x = 3 + 5y$

$x(y-2) = 5y + 3$

$x = \dfrac{5y+3}{y-2}$

$g^{-1}(y) = \dfrac{5y+3}{y-2}$

Domain of $g = \{x \mid x \neq 5\}$

Range of g = Domain of $g^{-1} = \{y \mid y \neq 2\}$