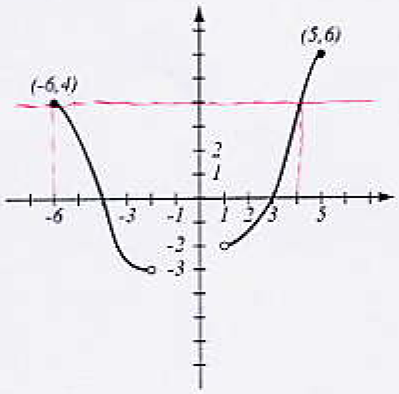
Spring '10/MAT 140/Joysheet 6 **Name:** Solution *Show all your work.*

1. (14pts) Use the graph of the function f at right to answer the following questions.



a) What is the domain of f? $[-6,-2] \cup (1,5]$

b) What is the range of f? $(-3,6]$

c) Find $f(3)$ and $f(-1)$. $f(3)=0$ $f(-1)$ is not defined

d) What are the solutions of the equation $f(x) = 4$? $x=6$ and $x=4$

e) Find intervals where $f(x) < 0$.

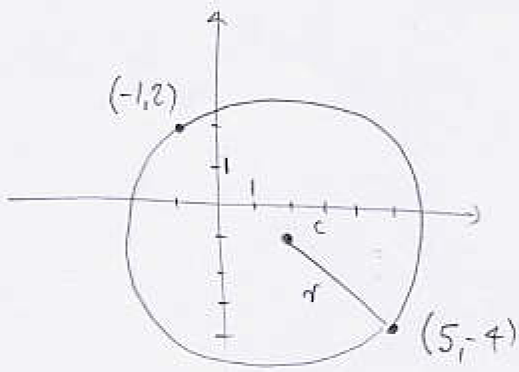
when graph is below x-axis

$(-4,-2) \cup (1,3)$

2. (11pts) Suppose $(-1,2)$ and $(5,-4)$ are endpoints of a diameter of a circle. Find the equation of the circle and draw the circle.

$m = \left(\frac{-1+5}{2}, \frac{2-4}{2}\right) = (2,-1)$

midpoint of the two points is the center of the circle



Radius $= \frac{1}{2}$ of distance between the points.

$d = \sqrt{(5-(-1))^2+(-4-2)^2} = \sqrt{6^2+6^2} = \sqrt{72} = 6\sqrt{2}$

$r = \frac{1}{2} \cdot d = 3\sqrt{2}$ Equation of circle: $(x-2)^2+(y+1)^2 = 18$

3. (10pts) Find the domain of $f(x) = \dfrac{\sqrt{3x+2}}{x^2 - 3x - 40}$. Write your answer in interval notation.

Must have: $3x+2 \geq 0$ $|-2$

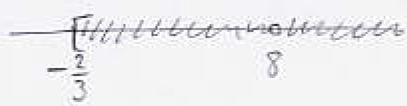
$3x \geq -2$ $|\div 3$

$x \geq -\frac{2}{3}$

Can't have $x^2-3x-40=0$

$(x-8)(x+5)=0$

$x = 8, -5$



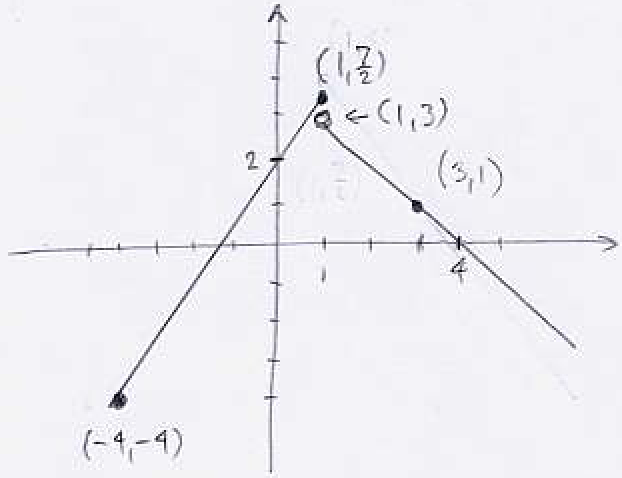
$[-\frac{2}{3}, 8) \cup (8, \infty)$

4. (9pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} \frac{3}{2}x + 2, & \text{if } -4 \le x \le 1 \\ -x + 4, & \text{if } +1 < x. \end{cases}$$

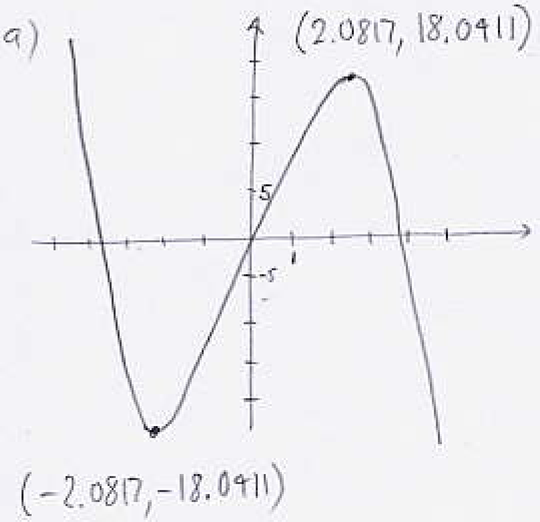
x	$\frac{3}{2}x+2$
1	$\frac{7}{2} = 3.5$
-4	-4

x	$-x+4$
1	3
3	1



5. (16pts) Let $f(x) = -x^3 + 13x$ (answer with 4 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.
b) Determine algebraically whether f is even, odd, or neither. Justify your answer further by examining the graph.
c) Find where f has a local minimum and maximum.
d) Find the intervals of increase and decrease.

a)



b) $f(-x) = -(-x)^3 + 13(-x)$
$= -(-x^3) - 13x$
$= x^3 - 13x = -f(x)$

f is odd, which can also be seen on the graph - it is symmetric w.r.t. the origin.

c) f has a local min. at $x = -2.0817$ with value $y = -18.0411$
f has a local max. at $x = 2.0817$ with value $y = 18.0411$

d) f is increasing on $(-2.0817, 2.0817)$
f is decreasing on $(-\infty, -2.0817) \cup (2.0817, \infty)$