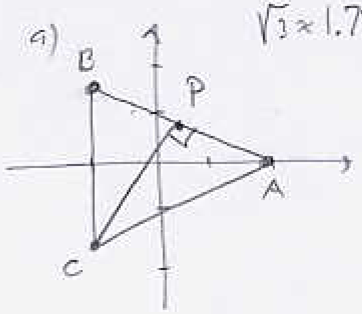
Spring '10/MAT 140/Joysheet 5 **Name:** Solution *Show all your work.*

1. (18pts) Let $A = \cancel{(0,2)}\ (2,0)$, $B = (-1, \sqrt{3})$, $C = (-1, -\sqrt{3})$.
a) Draw the picture.
b) Show algebraically that the triangle ABC is equilateral (all sides have equal length).
c) Use the Pythagorean theorem to show that the triangle APC is right, where P is the midpoint of AB.

a)



$\sqrt{3} \approx 1.73$

b)
$d(A,B) = \sqrt{(-1-2)^2 + (\sqrt{3}-0)^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}$ (4·3)
$d(B,C) = \sqrt{(-1-(-1))^2 + (\sqrt{3}-\sqrt{3})^2} = \sqrt{(2\sqrt{3})^2} = 2\sqrt{3}$
$d(C,A) = \sqrt{(2-(-1))^2 + (0-(-\sqrt{3}))^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}$

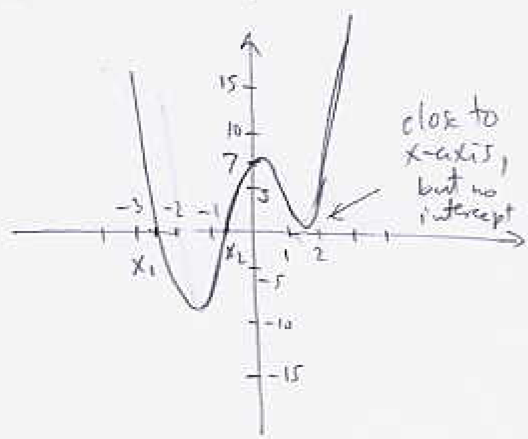
all are equal so it is an equilateral triangle

c) $P = \left(\frac{2+(-1)}{2}, \frac{0+\sqrt{3}}{2}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

check
$d(A,C)^2 \stackrel{?}{=} d(A,P)^2 + d(P,C)^2 \leftarrow$
$(2\sqrt{3})^2 \stackrel{?}{=} \sqrt{3}^2 + 3^2$
$12 \stackrel{?}{=} 3 + 9$
yes, so it is a right triangle.

$d(A,C) = 2\sqrt{3}$
$d(A,P) = \frac{1}{2} d(A,B) = \frac{1}{2} \cdot 2\sqrt{3} = \sqrt{3}$
$d(P,C) = \sqrt{\left(-1-\frac{1}{2}\right)^2 + \left(-\sqrt{3}-\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\left(-\frac{3}{2}\right)^2 + \left(-\frac{3}{2}\sqrt{3}\right)^2} = \sqrt{\frac{9}{4} + \frac{9}{4}\cdot 3}$
$= \sqrt{\frac{36}{4}} = 3$

2. (12pts) Use your calculator to accurately sketch the graph of $y = x^4 - 7x^2 + 3x + 7$. Draw the graph here, and indicate the viewing window. Find all the $x-$ and $y-$intercepts (accuracy: 4 decimal points).



y-int: 7

x-int: $x_1 = -2.6725$
$x_2 = -0.8431$

3. (8pts) Find the equation of the line (in form $y = mx + b$) that passes through points $(-1,-2)$ and $(2,3)$.

$$m = \frac{3-(-2)}{2-(-1)} = \frac{5}{3}$$

$$y-3 = \frac{5}{3}(x-2)$$

$$y = \frac{5}{3}x - \frac{10}{3} + 3$$

$$y = \frac{5}{3}x - \frac{1}{3}$$

4. (14pts) Find the equation of the line (in form $y = mx + b$) that is parallel to the line $2x - 3y = 7$, and passes through point $(-1,3)$. Draw both lines.

$$2x-3y=7$$

$$-3y = -2x+7 \quad |\div -3$$

$$y = \frac{2}{3}x - \frac{7}{3}$$

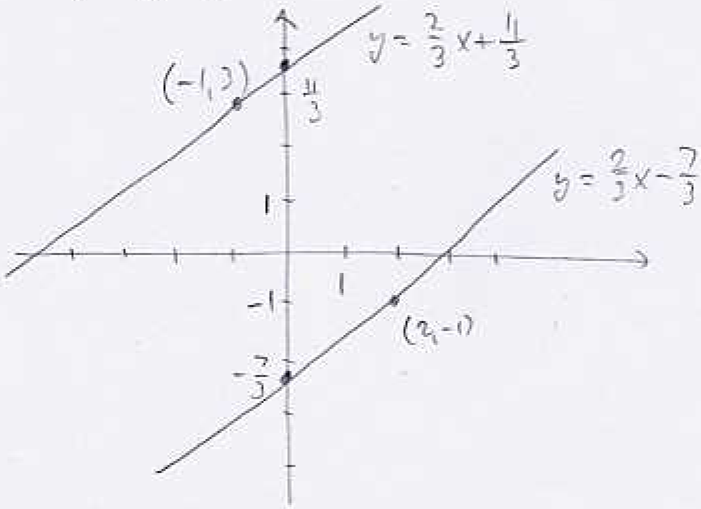
slope $= \frac{2}{3}$, so

slope of the parallel line is $\frac{2}{3}$

x	y
2	-1

$$y-3 = \frac{2}{3}(x-(-1))$$

$$y = \frac{2}{3}x + \frac{2}{3} + 3 = \frac{2}{3}x + \frac{11}{3}$$



5. (8pts) Show that the triangle APC from problem 1 is right using another method, by showing that lines AB and PC are perpendicular (show that the product of their slopes is -1).

$$\left.\begin{array}{l} A=(2,0) \\ B=(-1,\sqrt{3}) \end{array}\right\} \quad m_1 = \frac{\sqrt{3}-0}{-1-2} = -\frac{\sqrt{3}}{3}$$

$$\left.\begin{array}{l} P=\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right) \\ C=(-1,-\sqrt{3}) \end{array}\right\} \quad m_2 = \frac{-\sqrt{3}-\frac{\sqrt{3}}{2}}{-1-\frac{1}{2}} = \frac{-\frac{3}{2}\sqrt{3}}{-\frac{3}{2}} = \sqrt{3}$$

$$m_1 \cdot m_2 = -\frac{\sqrt{3}}{3}\cdot\sqrt{3} = -\frac{3}{3} = -1$$