Spring '10/MAT 140/Joysheet 4 **Name:** Solution *Show all your work.*

1. (23pts) Solve the equations.

$3x^2 - x = 5x + 11$ $|-5x, -11$

$3x^2 - 6x - 11 = 0$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 3 \cdot (-11)}}{2 \cdot 3}$ $168 = 4 \cdot 42$

$= \frac{6 \pm \sqrt{36 + 132}}{6} = \frac{6 \pm \sqrt{168}}{6}$

$= \frac{6 \pm 2\sqrt{42}}{6} = \frac{2(3 \pm \sqrt{42})}{6} = \frac{3 \pm \sqrt{42}}{3}$

$x^6 + 3x^3 = 40$

$x^6 + 3x^3 - 40 = 0$ $x^6 = (x^3)^2$, set $u = x^3$

$u^2 + 3u - 40 = 0$

$(u+8)(u-5) = 0$

$u = -8 \quad u = 5$

$x^3 = -8 \quad x^3 = 5$

$x = \sqrt[3]{-8} = -2 \quad x = \sqrt[3]{5}$

$x - 2\sqrt{11 - x} = 3$ $|+2\sqrt{11-x}, -3$

$x - 3 = 2\sqrt{11 - x}$ $|^2$

$x^2 - 6x + 9 = 4(11 - x)$

$x^2 - 6x + 9 = 44 - 4x$ $|+4x, -44$

$x^2 - 2x - 35 = 0$

$(x-7)(x+5) = 0$

$x = 7, -5$

Only $x = 7$ is the solution

Test: $7 - 2\sqrt{11-7} \stackrel{?}{=} 3$

$7 - 2 \cdot 2 \stackrel{?}{=} 3$ yes

$-5 - 2\sqrt{11+5} \stackrel{?}{=} 3$

$-5 - 2 \cdot 4 \stackrel{?}{=} 3$

$-13 \stackrel{?}{=} 3$ no

2. (6pts) Solve by completing the square.

$x^2 + 16x = 19$ $|+8^2$ $(8 = \frac{16}{2})$

$x^2 + 2 \cdot x \cdot 8 + 8^2 = 19 + 8^2$

$(x+8)^2 = 83$

$x + 8 = \pm\sqrt{83}$

$x = -8 \pm \sqrt{83}$

3. (4pts) Solve the equation.

$|4x+1|=7$

$4x+1=7$ or $4x+1=-7$

$4x=6$ $\quad 4x=-8$

$x=\frac{6}{4}=\frac{3}{2}$ $\quad x=-\frac{8}{4}=-2$

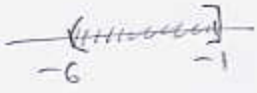
$x=\frac{3}{2}, -2$

4. (12pts) Solve the inequalities. Draw your solution and write it in interval form.

$5 \le 3-2x < 15$ $\quad |-3$

$2 \le -2x < 12$ $\quad |\div -2$

$-1 \ge x > -6$

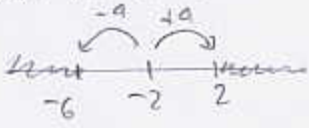


$(-6,-1]$

$|3x+2| \ge 4$

$|3x-(-2)| \ge 4$

dist from $3x$ to $-2 \ge 4$



$3x \le -6$ or $3x \ge 2$

$x \le -2$ or $x \ge \frac{2}{3}$

$(-\infty,-2] \cup [\frac{2}{3},\infty)$

5. (15pts) A landscaper has 72 cubic feet of stones that he will use to fill two circular areas to a depth of $\frac{1}{2}$ foot . If the bigger circular area has a radius that is 1 foot larger than the radius of the smaller one, what are the radii of the areas?

volume V_1 $= \pi(x+1)^2 \cdot \frac{1}{2}$

volume V_2 $\pi x^2 \cdot \frac{1}{2}$

$V_1 + V_2 = 72$

$\frac{1}{2}\pi(x+1)^2 + \frac{1}{2}\pi x^2 = 72 \quad |\cdot 2$

$\pi(x^2+2x+1) + \pi x^2 = 144$

$2\pi x^2 + 2\pi x + \pi - 144 = 0$

$a \quad b \quad c$

$$x = \frac{-2\pi \pm \sqrt{(2\pi)^2 - 4\cdot 2\pi\cdot(\pi-144)}}{2\cdot 2\pi}$$

$$= \frac{-6.28 \pm \sqrt{3579.636319}}{12.56} \approx 4.2611$$

($-6-\sqrt{}$ is clearly negative, does not fit context)

Radius of smaller circular area $= 4.2611$

bigger $= 5.2611$