Spring '10/MAT 140/Joysheet 2 **Name:** Solution *Show all your work.*

1. (16pts) Simplify.

$$\frac{x-1}{x^2+2x-15} - \frac{2x-1}{3x^2+14x-5} = \frac{x-1}{(x+5)(x-3)} - \frac{2x-1}{(3x-1)(x+5)} =$$

prod = −15, sum = 2: 5, −3

prod = −15, sum = 14: 15, −1

$3x^2+15x-x-5$

$3x(x+5)-(x+5)$

$(3x-1)(x+5)$

$$= \frac{(x-1)(3x-1) - (2x-1)(x-3)}{(x+5)(x-3)(3x-1)} = \frac{3x^2-4x+1-(2x^2-7x+3)}{(x+5)(x-3)(3x-1)} = \frac{x^2+3x-2}{(x+5)(x-3)(3x-1)}$$

prod = −2: 1, −2; −1, 2

sum = 3: no; no

numerator does not factor

$$\frac{3+\frac{3}{x-4}}{x^2+4x+3} \cdot \frac{x^2-16}{x-3} = \frac{\frac{3(x-4)+3}{x-4}}{(x+1)(x+3)} \cdot \frac{(x-4)(x+4)}{x-3}$$

prod = 3, sum = 4: 1, 3

$$= \frac{3x-9}{x-4} \cdot \frac{1}{(x+1)(x+3)} \cdot \frac{(x-4)(x+4)}{x-3}$$

$$= \frac{3(x-3)(x+4)}{(x+1)(x+3)(x-3)} = \frac{3(x+4)}{(x+1)(x+3)}$$

2. (9pts) Simplify, showing intermediate steps.

$\sqrt{72} = \sqrt{9 \cdot 8} = 3\sqrt{4 \cdot 2} = 3 \cdot 2\sqrt{2} = 6\sqrt{2}$

$81^{\frac{3}{4}} = \left(\sqrt[4]{81}\right)^3 = 3^3 = 27$

$(-32)^{\frac{2}{5}} = \left(\sqrt[5]{-32}\right)^2 = (-2)^2 = 4$

3. (14pts) Simplify. Express answers in terms of positive exponents.

$\sqrt[4]{48x^{16}y^{10}} = \sqrt[4]{16\cdot 3\cdot (x^4)^4 (y^2)^4 \cdot y^2} = 2\sqrt[4]{3}\cdot x^4 \cdot y^2 \sqrt[4]{y^2}$

$y^8 \cdot y^2$

$= 2x^4y^2\sqrt[4]{3y^2}$

$$\frac{x^{\frac{3}{4}}\left(25x^{\frac{5}{4}}y^{-\frac{2}{3}}\right)^{\frac{3}{2}}}{10\left(36y^{\frac{8}{3}}\right)^{-\frac{1}{2}}} = \frac{x^{\frac{3}{4}}\cdot(\sqrt{25})^3\cdot x^{\frac{5}{4}\cdot\frac{3}{2}}\, y^{-\frac{2}{3}\cdot\frac{3}{2}}}{10\cdot 36^{-\frac{1}{2}}\, y^{\frac{8}{3}\cdot(-\frac{1}{2})}} = \frac{x^{\frac{3}{4}}\cdot 125\cdot x^{\frac{15}{8}}\, y^{-1}\cdot 36^{\frac{1}{2}}}{2\cancel{10}\cdot y^{-\frac{4}{3}}}$$

$$= \frac{25\cdot x^{\frac{3}{4}+\frac{15}{8}-1-(-\frac{4}{3})}\, y^{-1-(-\frac{4}{3})}\cdot \cancel{6}^{3}}{\cancel{2}\,1} = 75x^{\frac{21}{8}}y^{\frac{1}{3}}$$

4. (6pts) Rationalize the denominator.

$$\frac{3\sqrt{7}-1}{5+\sqrt{7}} = \frac{3\sqrt{7}-1}{5+\sqrt{7}}\cdot\frac{5-\sqrt{7}}{5-\sqrt{7}} = \frac{15\sqrt{7}-3\sqrt{7}^2-5+\sqrt{7}}{5^2-\sqrt{7}^2} = \frac{16\sqrt{7}-21-5}{25-7} = \frac{16\sqrt{7}-26}{18}$$

$$= \frac{2(8\sqrt{7}-13)}{\cancel{18}\,9} = \frac{8\sqrt{7}-13}{9}$$

5. (10pts) Simplify.

$(4+i)(3-5i) = 12-20i+3i-5\underbrace{i^2}_{=-1} = 17-17i$

$$\frac{3-4i}{4+3i} = \frac{3-4i}{4+3i}\cdot\frac{4-3i}{4-3i} = \frac{12-9i-16i+12\underbrace{i^2}_{=-1}}{4^2-(3i)^2} = \frac{-25i}{16+9} = \frac{-25i}{25} = -i$$

6. (5pts) Simplify and justify your answer.

$i^{1002} = i^{4\cdot 250+2} = \underbrace{(i^4)}_{=1}{}^{250}\cdot i^2 = i^2 = -1$

$1002 \div 4 = 250$, rem 2

$1002 = 4\cdot 250 + 2$