1. (4pts) Convert to scientific notation or a decimal number:

$34789.31 =$ $3.989 \times 10^{-4} =$

2. (14pts) Use formulas to expand:

$(3y + 7)^2 =$

$(5u + v^2)(5u - v^2) =$

$(x + 5)^3 =$

3. (13pts) Simplify, showing intermediate steps.

$\sqrt{24} =$ $\sqrt[3]{-432} =$

$16^{\frac{5}{4}} =$ $\sqrt{75x^9y^6} =$

4. (8pts) Simplify.

$$\frac{4x+3}{x^2+3x-28} - \frac{2}{x^2-16} =$$

5. (7pts) Simplify and write the answer so all exponents are positive:

$$\frac{4(9x^{-4}y)^{\frac{3}{2}}}{(2x^{-5}y^{\frac{5}{8}})^4} =$$

6. (6pts) Rationalize the denominator.

$$\frac{3+2\sqrt{5}}{4+\sqrt{5}}$$

7. (11pts) Put the complex number into form $a+bi$.

$$\frac{3+2i}{3-5i}$$

(explain) $i^{171} =$

8. (13pts) Solve the equations.

$$\frac{2x+1}{5} + 4 = \frac{3x+2}{6} - \frac{2x-7}{10}$$

$$\frac{3x-4}{x-2} + 4 = \frac{2}{x-2}$$

9. (12pts) How many liters of pure anti-freeze needs to be added to 5 liters of a 15% solution of antifreeze in order to get a 50% solution? Write down the meaning of the variable you use.

10. (12pts) Ashley deposited some money in an account yielding 7%, and then deposited the same amount, and $700 extra, in an account yielding 11%. How much did she deposit in each account, if after 9 months the interest earned from both accounts totaled $408.75? Write down the meaning of the variable you use.

Bonus (10pts) An oil tanker can be emptied by the main pump in 4 hours. An auxiliary pump can empty the tanker in 9 hours. If the main pump is started now, when should the auxiliary pump be turned on so that the tanker is emptied in exactly 3 hours? *(Hint: Consider what portion of the job is to be done by the auxiliary pump.)*

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.
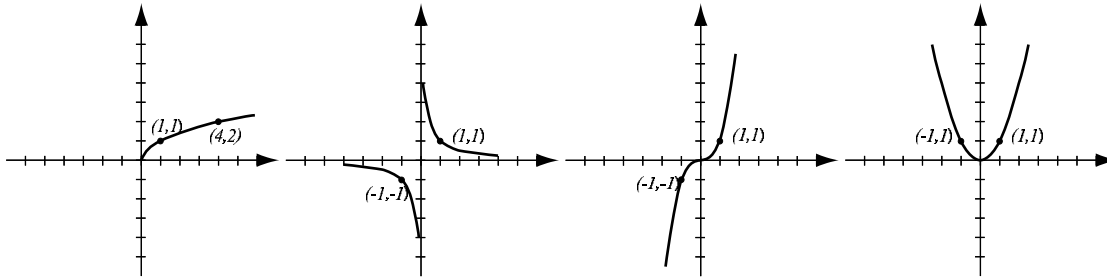
2. (11pts) Solve the inequalities and write the solution using interval notation:

$2x - 3 < 12$ $|x - 3| \geq 5$

3. (8pts) Solve the equation.

$3x^2 - 5x = 2x + 6$

4. (10pts) Solve the equation.

$x^4 - 4x^2 - 21 = 0$

5. (10pts) Use the graph of the function f at right to answer the following questions.
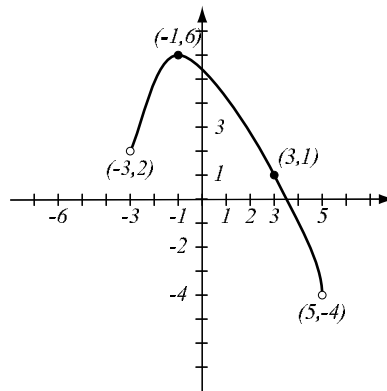
a) What is the domain of f?

b) What is the range of f?

c) Find $f(3)$ and $f(-3)$.

d) What are the solutions of the equation $f(x) = -1$?

e) Find all x for which $f(x) \geq 0$.

6. (8pts) Find the equation of the line (in form $y = mx + b$) that is perpendicular to the line $3x - 4y = 7$ and passes through the point $(-1, 1)$.

7. (9pts) Below is an equation of a circle. Find the center and radius of the circle and draw the circle.

$$x^2 + y^2 + 6x - 2y + 6 = 0$$

8. (22pts) Let $f(x) = x^4 - 5x^2 - 9$ (answer with 4 decimal points accuracy).

a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.

b) Determine algebraically whether f is even, odd, or neither. Justify your answer further by examining the graph.

c) Find the x- and y-intercepts.

d) Find where f has a local minimum and maximum.

e) Find the intervals of increase and decrease.

9. (6pts) Find the domain of the function $g(x) = \sqrt{3 - 4x}$.

10. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 2x - 1, & \text{if } -4 \le x \le 2 \\ -x + 3, & \text{if } 2 < x < 6. \end{cases}$$

Bonus (10pts) Find all points on the line $y = x - 3$ whose distance to the point $(1, 2)$ equals 5. Draw a picture. *(Hint: Set up an equation involving the distance between a generic point (x, y) and the point $(1, 2)$. Then use the fact that the point (x, y) has to be on the line.)*

1. (20pts) Let $f(x) = \sqrt{3x + 16}$, $g(x) = \dfrac{1}{x} + 4$.

Find the following (simplify where possible):

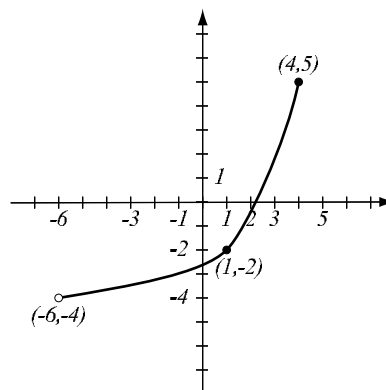$\dfrac{f}{g}(2) =$ $(f \cdot g)(x) =$

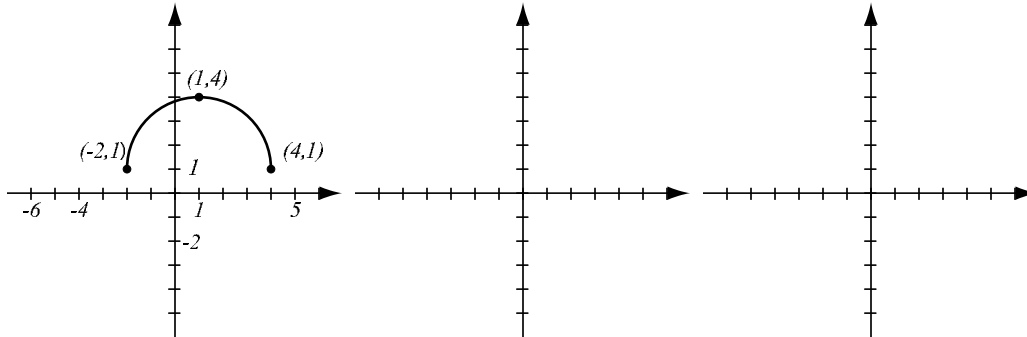$(g \circ f)(0)$ $(f \circ g)(x) =$

The domain of $(f - g)(x)$

2. (7pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points.

3. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(x-1)+3$ and $f(2x)$ and label all the relevant points.

4. (15pts) The quadratic function $f(x) = -x^2 - 3x + 10$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

d) What is the range of the function?

5. (22pts) Consider the polynomial $f(x) = x^4 - 9x^3 + 18x^2$.
a) Find the y- and x-intercepts algebraically. What are the multiplicities of the zeroes of f?
b) Use your calculator to draw the graph of the function (on paper!).
c) Find all the turning points (4 decimal points accuracy).
d) Describe the end behavior of f.

6. (10pts) Let $f(x) = (3x + 2)^3$.
a) Find the formula for f^{-1}.
b) Show that $(f \circ f^{-1})(y) = y$.

7. (16pts) Farmer Tom has 5000 meters of fencing. He would like to enclose a rectangular area and divide it in half with a fence so that the area is the largest possible. Find the dimensions of the enclosure that will give the greatest area. What is the greatest area?
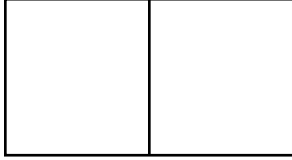
Bonus. (10pts) Find the point on the line $y = 5 - 3x$ that is closest to the point $(-1, 3)$. Draw a picture. Hints: Set up the expression for the distance d between a generic point (x, y) and the point $(-1, 3)$. Then express d only in terms of x, and minimize d^2 (you will need to simplify d^2).

1. (8pts) Evaluate without using the calculator:

$$\log_8 64 = \qquad\qquad \log_3 \tfrac{1}{9} = \qquad\qquad \ln \sqrt[3]{e} = \qquad\qquad \log_4 32 =$$

2. (4pts) Use your calculator to find $\log_{13} 0.13$ with accuracy 4 decimal places. Show how you obtained your number.

3. (12pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_7 \left(\frac{49y^3}{\sqrt[8]{x^3}} \right) =$$

$$\log_9 ((x^2 - 6x + 9)(x^2 + 10x + 25)) =$$

4. (12pts) Write as a single logarithm. Simplify if possible.

$$3\log(6x^2) + 2\log(3x^4) =$$

$$\log_2(x + 3) + \log_2(x - 7) - 2\log_2(x^2 - 4x - 21) =$$

5. (7pts) In November 1755, Lisbon (Portugal) was destroyed by an earthquake which released 8×10^{17} joules of energy. Find the magnitude of this earthquake using the Richter scale. (Recall that magnitude is given by $M = \frac{2}{3} \log \left(\frac{E}{E_0} \right)$, where $E_0 = 10^{4.4}$, the energy released by a reference earthquake.)

6. (7pts) How much should you invest in an account bearing 5.25%, compounded monthly, if you wish to have \$1,500 in four years?

7. (8pts) Draw the general shape of the graph for these functions. Indicate the x- and y-intercepts. What are the horizontal or vertical asymptotes of the graphs?

$f(x) = b^x$, $b > 1$ $\hspace{5cm}$ $f(x) = \log_b x$, $b > 1$.

Solve the equations.

8. (10pts) $2^{x^2+3x-8} = 16^{x+3}$

9. (12pts) $\log_2(x-1) - \log_2(x-3) = 3$

10. (10pts) $4^{x+2} = 5^{x+3}$

11. (10pts) Suppose you invest \$2,000 at a 3% interest rate, compounded continuously. How long will it take until your investment has value \$4,000? (Recall that $A = Pe^{rt}$.)

Bonus (10pts) The population of the Mushroomton is given by the formula $N(t) = 103e^{rt}$ (in thousands), where t is the number of years since 2003, and r is the growth rate.
a) If the population was 143,270 in 2006, find the growth rate r.
b) If the city continues to grow at the same rate, what will be its population in 2011?

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.
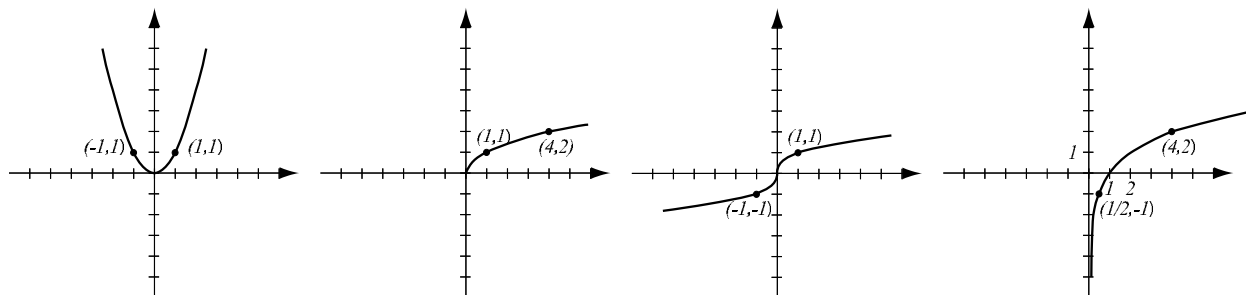
2. (10pts) Use the graph of the function f at right to answer the following questions.

a) What is the domain of f?

b) What is the range of f?

c) Find $f(6)$ and $f(-7)$.

d) What are the solutions
   of the equation $f(x) = 1$?

e) Find all x for which $f(x) \geq 0$.
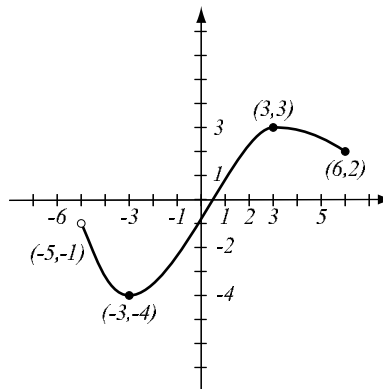
3. (7pts) Simplify and write the answer so all exponents are positive:

$$\frac{(3x^{-2}y)^3}{(6x^4y^{-2})^2} =$$

4. (6pts) Solve the equation.

$$\frac{3x-2}{4} + 3 = \frac{2x-1}{12} - \frac{x+5}{3}$$

5. (7pts) Solve the inequality and write the solution using interval notation:

$|x - 3| \le 6$

6. (10pts) Let $f(x) = x^2 - 3x + 5$, $g(x) = 2x - 1$.
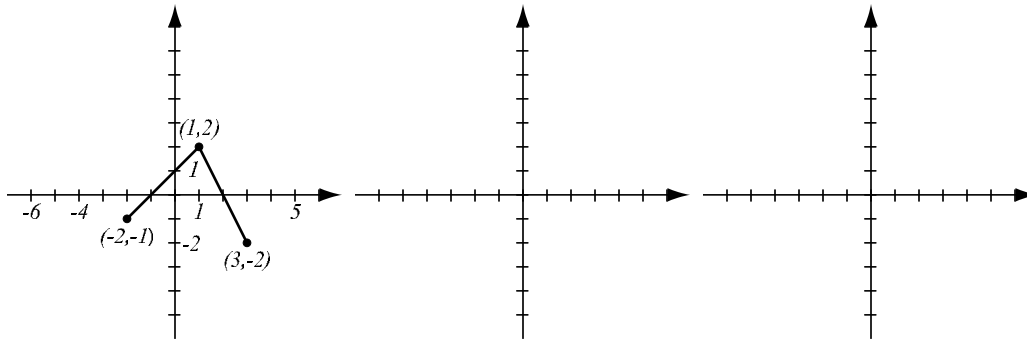
Find the following (simplify where possible):

$(f \cdot g)(x) =$ $(f \circ g)(x) =$

7. (8pts) Find the equation of the line (in form $y = mx + b$) that is parallel to the line $2x - 5y = 2$ and passes through the point $(-1, 1)$.

8. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x+2)$ and $2f(x)$ and label all the relevant points.

9. (14pts) The quadratic function $f(x) = 4x^2 - 8x - 5$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

10. (5pts) Find the domain of the function $g(x) = \dfrac{3}{\sqrt{5x - 4}}$.

11. (21pts) Consider the polynomial $f(x) = x^3 + 14x^2 + 49x$.
a) Find the y- and x-intercepts algebraically. What are the multiplicities of the zeroes of f?
b) Use your calculator to draw the graph of the function (on paper!).
c) Determine algebraically whether f is even, odd, or neither. Justify your answer further by examining the graph.
d) Find all the turning points (4 decimal points accuracy).
e) Describe the end behavior of f.

12. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_3\left(\frac{y^3}{27\sqrt{x^3}}\right) =$$

13. (6pts) Write as a single logarithm. Simplify if possible.

$$2\log_5(x-6) + 2\log_5(x+3) - \log_5(x^2 - 3x - 18) =$$

14. (10pts) Solve the equation.

$$5^{x^2+8x+7} = 125^{x-7}$$

15. (10pts) Suppose you invest \$2,000 at a 3% interest rate, compounded monthly. How long will it take until your investment has value \$4,000?

16. (12pts) How many milliliters of a 10% solution of sulphuric acid needs to be added to 3 milliliters of a 35% solution of sulphuric acid in order to get a 25% solution? Write down the meaning of the variable you use.

Bonus (10pts) Farmer Tom has 5000 meters of fencing. He would like to enclose a rectangular area and divide it in half with a fence so that the area is the largest possible. Find the dimensions of the enclosure that will give the greatest area. What is the greatest area?