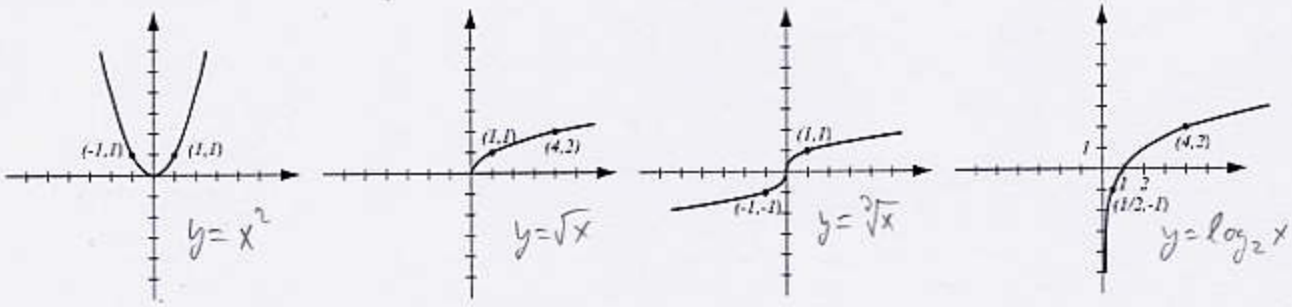
Spring '10/MAT 140/Final exam **Name:** Solution *Show all your work.*

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (10pts) Use the graph of the function f at right to answer the following questions.

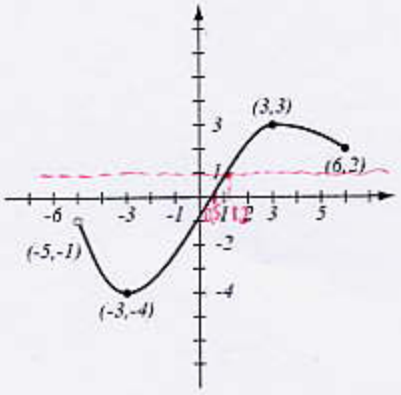
a) What is the domain of f? $(-5, 2]$

b) What is the range of f? $[-4, 3]$

c) Find $f(6)$ and $f(-7)$. $f(6) = 2$, $f(-7)$ is not defined

d) What are the solutions of the equation $f(x) = 1$? $x = 1.2$

e) Find all x for which $f(x) \geq 0$. $f(x) \geq 0$ on $[0.5, 6]$



3. (7pts) Simplify and write the answer so all exponents are positive:

$$\frac{(3x^{-2}y)^3}{(6x^4y^{-2})^2} = \frac{27x^{-6}y^3}{36x^8y^{-4}} = \frac{3x^{-6-8}y^{3-(-4)}}{4} = \frac{3x^{-14}y^7}{4} = \frac{3y^7}{4x^{14}}$$

4. (6pts) Solve the equation.

$$\frac{3x-2}{4}+3=\frac{2x-1}{12}-\frac{x+5}{3} \quad |\cdot 12$$

$$\frac{3x-2}{4}\cdot 12 + 36 = 2x-1-\frac{x+5}{3}\cdot 12$$

$$9x-6+36 = 2x-1-4x-20$$

$$9x+30 = -2x-21 \quad |+2x-30$$

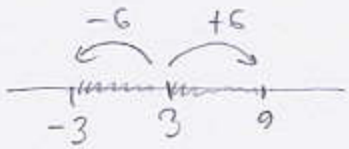
$$11x = -51$$

$$x = -\frac{51}{11}$$

5. (7pts) Solve the inequality and write the solution using interval notation:

$|x-3| \leq 6$

distance from x to $3 \leq 6$



$-3 \leq x \leq 9$

$[-3, 9]$

6. (10pts) Let $f(x) = x^2 - 3x + 5$, $g(x) = 2x - 1$.

Find the following (simplify where possible):

$(f \cdot g)(x) = f(x) \cdot g(x)$

$= (x^2 - 3x + 5)(2x-1)$

$= 2x^3 - 6x^2 + 10x - x^2 + 3x - 5$

$= 2x^3 - 7x^2 + 13x - 5$

$(f \circ g)(x) = f(g(x))$

$= f(2x-1) = (2x-1)^2 - 3(2x-1) + 5$

$= 4x^2 - 4x + 1 - 6x + 3 + 5$

$= 4x^2 - 10x + 9$

7. (8pts) Find the equation of the line (in form $y = mx + b$) that is parallel to the line $2x - 5y = 2$ and passes through the point $(-1, 1)$.

$2x - 5y = 2$

$2x - 2 = 5y$

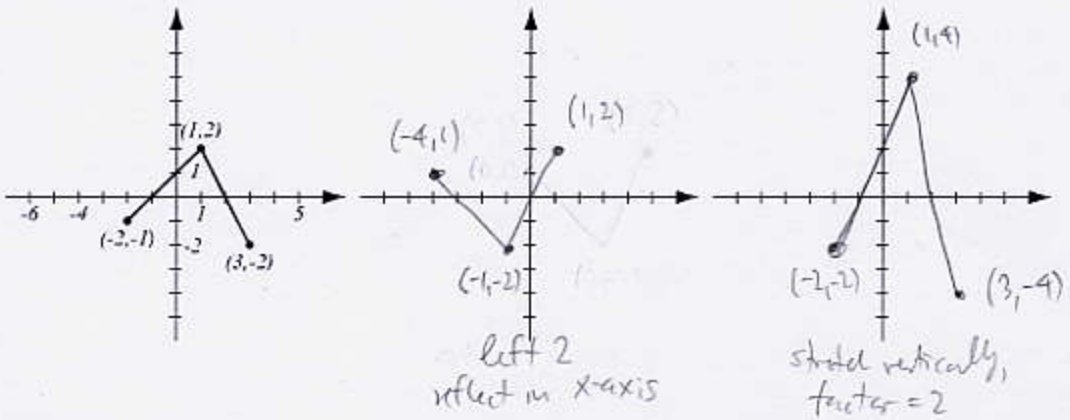
$y = \frac{2x-2}{5} = \frac{2}{5}x - \frac{2}{5}$

The line is parallel to $\frac{2}{5}x - \frac{2}{5}$, so it has the same slope, $\frac{2}{5}$.

$y - 1 = \frac{2}{5}(x - (-1))$

$y = \frac{2}{5}x + \frac{2}{5} + 1 = \frac{2}{5}x + \frac{7}{5}$

8. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x+2)$ and $2f(x)$ and label all the relevant points.



9. (14pts) The quadratic function $f(x) = 4x^2 - 8x - 5$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

a) y-int: $f(0) = -5$

x-int: $4x^2 - 8x - 5 = 0$

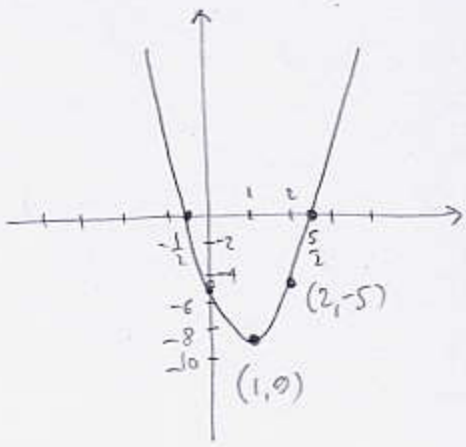
$x = \frac{8 \pm \sqrt{64 - 4 \cdot 4 \cdot (-5)}}{2 \cdot 4}$

$= \frac{8 \pm \sqrt{144}}{8} = \frac{8 \pm 12}{8}$

$= \frac{20}{8}, -\frac{4}{8} = \frac{5}{2}, -\frac{1}{2}$

b) $x = -\frac{b}{2a} = -\frac{-8}{2 \cdot 4} = 1$

$y = 4 \cdot 1^2 - 8 \cdot 1 - 5 = -9$

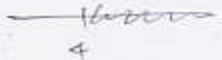


10. (5pts) Find the domain of the function $g(x) = \dfrac{3}{\sqrt{5x-4}}$.

Must have $5x-4>0$

$5x>4$

$x>\frac{4}{5}$



Domain: $(\frac{4}{5}, \infty)$

11. (21pts) Consider the polynomial $f(x) = x^3 + 14x^2 + 49x$.
a) Find the y- and x-intercepts algebraically. What are the multiplicities of the zeroes of f?
b) Use your calculator to draw the graph of the function (on paper!).
c) Determine algebraically whether f is even, odd, or neither. Justify your answer further by examining the graph.
d) Find all the turning points (4 decimal points accuracy).
e) Describe the end behavior of f.

a) $x^3+14x^2+49x=0$

$x(x^2+14x+49)=0$

$x(x+7)^2=0$

$x=0, -7$

multiplicity: 1, 2

c) $f(-x)=(-x)^3+14(-x)^2+49(-x)$

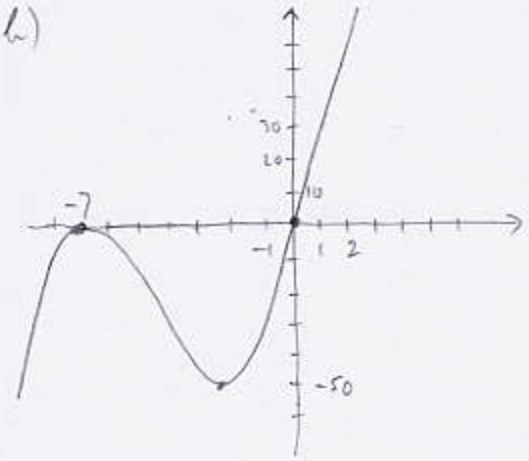
$=-x^3+14x^2-49x$

not equal to $f(x)$ nor $-f(x)$

so neither

Can see on the graph - there is no symmetry, either w.r.t. y-axis or the origin

b)



d) f has a local max at $x=-7$ whose value is $y=0$

f has a local min at $x=-2.333$ whose value is $y=-50.8148$

e) f behaves like x^3 for large $|x|$.

12. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_3\left(\frac{y^3}{27\sqrt{x^3}}\right) =$ $\log_3 y^3 - \log_3(27x^{\frac{3}{2}}) = 3\log_3 y - (\underbrace{\log_3 27}_{=3} + \log_3 x^{\frac{3}{2}})$

$= 3\log_3 y - 3 - \frac{3}{2}\log_3 x$

$x^{\frac{3}{2}}$

13. (6pts) Write as a single logarithm. Simplify if possible.

$2\log_5(x-6) + 2\log_5(x+3) - \log_5(x^2-3x-18) =$ $\log_5(x-6)^2 + \log_5(x+3)^2 - \log_5(x^2-3x-18)$ $(x-6)(x+3)$

$= \log_5 \frac{(x-6)^2(x+3)^2}{(x-6)(x+3)} = \log_5((x-6)(x+3))$

14. (10pts) Solve the equation.

$5^{x^2-8x+7} = 125^{x-7}$ $125 = 5^3$

$5^{x^2-8x+7} = (5^3)^{x-7}$

$5^{x^2-8x+7} = 5^{3x-21}$

$x^2-8x+7 = 3x-21 \quad | -3x+21$

$x^2-11x+28 = 0$

$(x-4)(x-7) = 0$

$x = 4$ or 7

15. (10pts) Suppose you invest \$2,000 at a 3% interest rate, compounded monthly. How long will it take until your investment has value \$4,000?

$4000 = 2000\left(1+\frac{0.03}{12}\right)^{12t} \quad | \div 2000$

$2 = (1.0025)^{12t} \quad | \ln$

$\ln 2 = \ln(1.0025)^{12t}$

$\ln 2 = 12t \ln(1.0025) \quad | \div 12\ln(1.0025)$

$t = \frac{\ln 2}{12\ln(1.0025)} = 23.1338$ years

16. (12pts) How many milliliters of a 10% solution of sulphuric acid needs to be added to 3 milliliters of a 35% solution of sulphuric acid in order to get a 25% solution? Write down the meaning of the variable you use.

x = amount of 10% solution, in ml

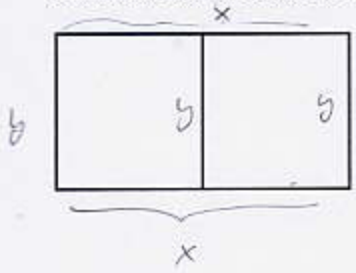
$$0.1x + 0.35 \cdot 3 = 0.25(x+3)$$

$$0.1x + 1.05 = 0.25x + 0.75 \quad | -0.1x - 0.75$$

$$0.3 = 0.15x$$

$$x = \frac{0.3}{0.15} = 2 \text{ ml}$$

Bonus (10pts) Farmer Tom has 5000 meters of fencing. He would like to enclose a rectangular area and divide it in half with a fence so that the area is the largest possible. Find the dimensions of the enclosure that will give the greatest area. What is the greatest area?



$$2x + 3y = 5000$$

$$3y = 5000 - 2x \quad | \div 3$$

$$y = \frac{5000 - 2x}{3}$$

$$A = xy = x \cdot \frac{5000 - 2x}{3} = \frac{1}{3}(-2x^2 + 5000x)$$

$$x = -\frac{5000}{2 \cdot (-2)} = \frac{5000}{4} = 1250$$

$$y = \frac{5000 - 2 \cdot 1250}{3} = \frac{2500}{3} = 833.33\ldots$$

$$A = 1250 \cdot \frac{2500}{3} = \frac{3125000}{3}$$

$$= 1041{,}666.67 \text{ m}^2$$