Spring '10/MAT 140/Exam 4 Name: Solution *Show all your work.*

1. (8pts) Evaluate without using the calculator:

$\log_8 64 = 2$ $\log_3 \frac{1}{9} = -2$ $\ln \sqrt[3]{e} = \frac{1}{3}$ $\log_4 32 = \frac{5}{2}$

$8^? = 64$ $3^? = \frac{1}{9}$ $3^2 = 9$ $\sqrt[3]{e} = e^{\frac{1}{3}} = e^?$ $4^? = 32$ $2^5 = 32$ $2 = 4^{\frac{1}{2}}$ $(4^{\frac{1}{2}})^5 = 4^{\frac{5}{2}}$

2. (4pts) Use your calculator to find $\log_{13} 0.13$ with accuracy 4 decimal places. Show how you obtained your number.

$$\log_{13} 0.13 = \frac{\ln 0.13}{\ln 13} = -0.7954$$

3. (12pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_7 \left(\frac{49y^3}{\sqrt[8]{x^3}} \right) = \log_7 49 + \log_7 y^3 - \log_7 x^{\frac{3}{8}}$$
$$= 2 + 3\log_7 7 - \frac{3}{8}\log_7 x$$

$$\log_9((x^2 - 6x + 9)(x^2 + 10x + 25)) = \log_9 (x^2 - 6x + 9) + \log_9 (x^2 + 10x + 25)$$
$$= \log_9 (x-3)^2 + \log_9 (x+5)^2$$
$$= 2\log_9 (x-3) + 2\log_9 (x+5)$$

4. (12pts) Write as a single logarithm. Simplify if possible.

$$3\log(6x^2) + 2\log(3x^4) = \log(6x^2)^3 + \log(3x^4)^2$$
$$= \log(216x^6) + \log(9x^8) = \log(216x^6 \cdot 9x^8) = \log(1944x^{14})$$

$$\log_2(x+3) + \log_2(x-7) - 2\log_2(x^2 - 4x - 21) = \log_2 \left((x+3)(x-7)\right) - \log_2 (x^2 - 4x - 21)^2$$

$$= \log_2 \frac{(x+3)(x-7)}{((x+3)(x-7))^2} = \log_2 \frac{\cancel{(x+3)}\cancel{(x-7)}}{(x+3)^{\cancel{2}}(x-7)^{\cancel{2}}} = \log_2 \frac{1}{(x+3)(x-7)}$$

5. (7pts) In November 1755, Lisbon (Portugal) was destroyed by an earthquake which released 8×10^{17} joules of energy. Find the magnitude of this earthquake using the Richter scale. (Recall that magnitude is given by $M = \frac{2}{3}\log\left(\frac{E}{E_0}\right)$, where $E_0 = 10^{4.4}$, the energy released by a reference earthquake.)

$$M = \frac{2}{3}\log\left(\frac{8\times 10^{17}}{10^{4.4}}\right) = \frac{2}{3}\log\left(8\times 10^{12.6}\right) = \frac{2}{3}\cdot 13.5031$$

$$= 9.0021 \text{ on Richter scale}$$

6. (7pts) How much should you invest in an account bearing 5.25%, compounded monthly, if you wish to have \$1,500 in four years?

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$1500 = P\left(1+\frac{0.0525}{12}\right)^{12\cdot 4}$$

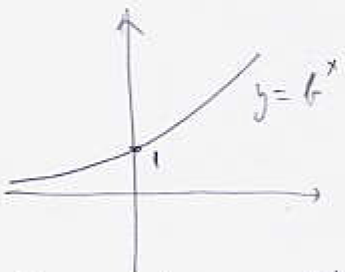
$$1500 = P(1.004375)^{48}$$

$$1500 = P\cdot 1.233\ldots$$

$$P = \frac{1500}{1.233\ldots} = 1216.43$$

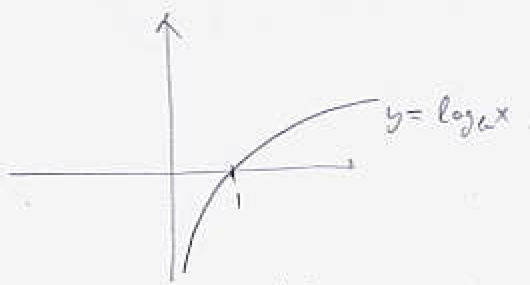
7. (8pts) Draw the general shape of the graph for these functions. Indicate the x- and y-intercepts. What are the horizontal or vertical asymptotes of the graphs?

$f(x) = b^x$, $b > 1$



horizontal asymptote: x-axis

$f(x) = \log_b x$, $b > 1$.



Vertical asymptote: y-axis

Solve the equations.

8. (10pts) $2^{x^2+3x-8} = 16^{x+3}$

$2^{x^2+3x-8} = (2^4)^{x+3}$

$2^{x^2+3x-8} = 2^{4x+12}$

$x^2+3x-8 = 4x+12 \quad | -4x-12$

$x^2-x-20=0$

$(x-5)(x+4)=0$

$x=-4, 5$

9. (12pts) $\log_2(x-1) - \log_2(x-3) = 3$

$\log_2(x-1) - \log_2(x-3) = 3$

$\log_2 \frac{x-1}{x-3} = 3$

cancel

$2^{\log_2 \frac{x-1}{x-3}} = 2^3$

$\frac{x-1}{x-3} = 8$

$x-1 = 8(x-3)$

$x-1 = 8x-24$

$23 = 7x$

$x = \frac{23}{7}$

Check:

$\log\left(\frac{23}{7}-1\right) - \log_2\left(\frac{23}{7}-3\right) \stackrel{?}{=} 3$

$\log_2 \frac{16}{7} - \log_2 \frac{2}{7} \stackrel{?}{=} 3$

$\log_2 \frac{\frac{16}{7}}{\frac{2}{7}} = \log_2 8 = 3$

Correct.

10. (10pts) $4^{x+2} = 5^{x+3}$

$4^{x+2} = 5^{x+3} \quad | \ln$

$\ln 4^{x+2} = \ln 5^{x+3}$

$(x+2)\ln 4 = (x+3)\ln 5$

$x\ln 4 + 2\ln 4 = x\ln 5 + 3\ln 5$

$x\ln 4 - x\ln 5 = 3\ln 5 - 2\ln 4$

$x(\ln 4 - \ln 5) = 3\ln 5 - 4\ln 4$

$x = \frac{3\ln 5 - 2\ln 4}{\ln 4 - \ln 5} \approx -9.2126$

11. (10pts) Suppose you invest \$2,000 at a 3% interest rate, compounded continuously. How long will it take until your investment has value \$4,000? (Recall that $A = Pe^{rt}$.)

$A = Pe^{rt}$

$4000 = 2000e^{0.03t} \quad | \div 2000$

$2 = e^{0.03t} \quad | \ln$

$\ln 2 = \ln e^{0.03t}$

$\ln 2 = 0.03t$

$t = \dfrac{\ln 2}{0.03} = 23.1049$ years

Bonus (10pts) The population of the Mushroomton is given by the formula $N(t) = 103e^{rt}$ (in thousands), where t is the number of years since 2003, and r is the growth rate.
a) If the population was 143,270 in 2006, find the growth rate r.
b) If the city continues to grow at the same rate, what will be its population in 2011?

a) $t=3$

$143.270 = 103e^{r \cdot 3} \quad | \div 103$

$1.39\ldots = e^{r \cdot 3} \quad | \ln$

$\ln(1.39097\ldots) = 3r \quad | \div 3$

$r = \dfrac{\ln(1.39097)}{3} = 0.11$

b) $N(8 = 103e^{8 \cdot 0.11}$

$= 103 \cdot e^{0.88}$

$= 103 \cdot 2.4109$

$= 248.323$

Approximately 248,323 people in 2011