Spring '10/MAT 140/Exam 3 **Name:** Solution *Show all your work.*

1. (20pts) Let $f(x) = \sqrt{3x+16}$, $g(x) = \frac{1}{x} + 4$.

Find the following (simplify where possible):

$\frac{f}{g}(2) = \frac{f(2)}{g(2)} = \frac{\sqrt{3\cdot 2+16}}{\frac{1}{2}+4} = \frac{\sqrt{22}}{\frac{9}{2}} = \frac{2\sqrt{22}}{9}$

$(f \cdot g)(x) = \sqrt{3x+16}\left(\frac{1}{x}+4\right)$

$(g \circ f)(0) = g(f(0)) = g\left(\sqrt{3\cdot 0+16}\right)$

$= g(4) = \frac{1}{4}+4 = \frac{17}{4}$

$(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x}+4\right)$

$= \sqrt{3\left(\frac{1}{x}+4\right)+16} = \sqrt{\frac{3}{x}+28}$

The domain of $(f-g)(x)$ = domain of f $\cap$ domain of g $= \left[-\frac{16}{3}, 0\right) \cup (0, \infty)$

$\sqrt{3x+16}$ $\quad$ $\frac{1}{x}+4$

$3x+16 \geq 0$ $\quad$ $x \neq 0$

$3x \geq -16$

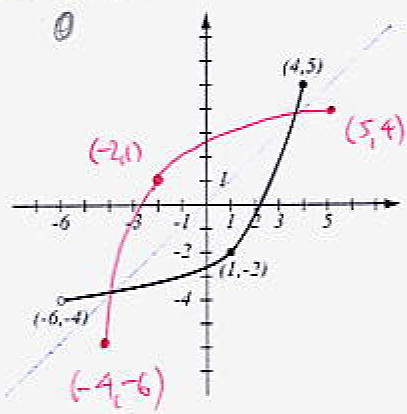
$x \geq -\frac{16}{3}$

$-\frac{16}{3}$ $\quad$ 0

2. (7pts) The graph of a function f is given.

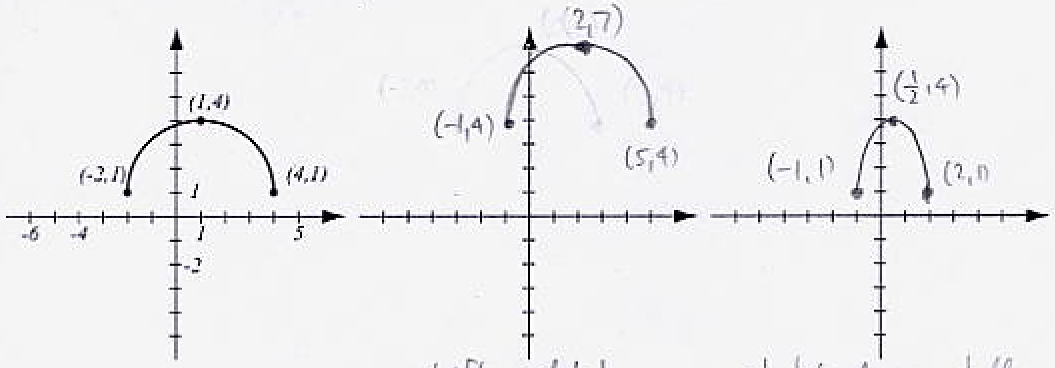
a) Is this function one-to-one? Justify.

b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points.



a) Yes, because it passes the horizontal line test

3. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(x-1)+3$ and $f(2x)$ and label all the relevant points.



shift right 1,
up 3

stretch horizontally,
factor $= \frac{1}{2}$

4. (15pts) The quadratic function $f(x) = \cancel{-x^2 - 3x + 10}$ is given. Do the following without using the calculator.
$-x^2 - 4x + 12$

a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.
d) What is the range of the function?

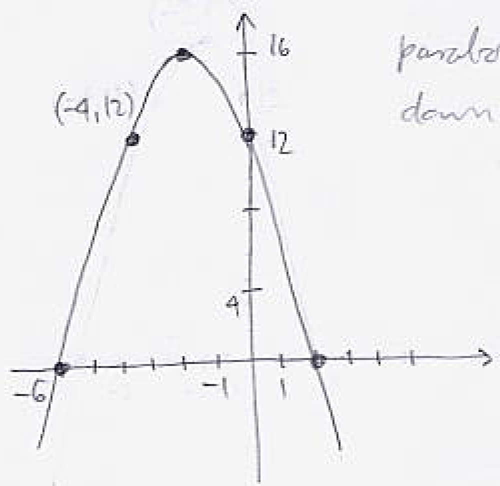
a) y-int: $f(0) = 12$

x-int: $-x^2 - 4x + 12 = 0$

$x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$

$x = -6, 2$



parabola opens down as $a<0$

b) $h = -\frac{-4}{2\cdot(-1)} = -2$

$k = -(-2)^2 - 4(-2) + 12$

$= -4 + 8 + 12 = 16$

d) Range $= (-\infty, 16]$

5. (22pts) Consider the polynomial $f(x) = x^4 - 9x^3 + 18x^2$.
a) Find the y- and x-intercepts algebraically. What are the multiplicities of the zeroes of f?
b) Use your calculator to draw the graph of the function (on paper!).
c) Find all the turning points (4 decimal points accuracy).
d) Describe the end behavior of f.

a) $f(0)=0$

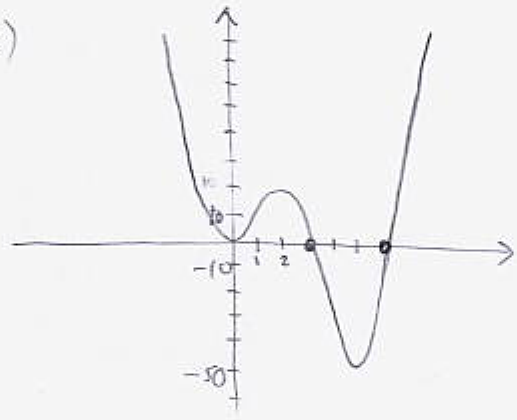
$x^4 - 9x^3 + 18x^2 = 0$

$x^2(x^2 - 9x + 18) = 0$

$x^2(x-6)(x-3) = 0$

zero	multiplicity
0	2
6	1
3	1

b)



c) f has a local min at $x=0$ whose value is $y=0$

" $x=4.9212$ " $y=-50.1944$

f has a local max at $x=1.8288$ whose value is $y=16.3390$

d) behaves like x^4

6. (10pts) Let $f(x) = (3x+2)^3$.
a) Find the formula for f^{-1}.
b) Show that $(f \circ f^{-1})(y) = y$.

a) $y = (3x+2)^3 \quad | \sqrt[3]{\ }$

$\sqrt[3]{y} = 3x+2 \quad | -2$

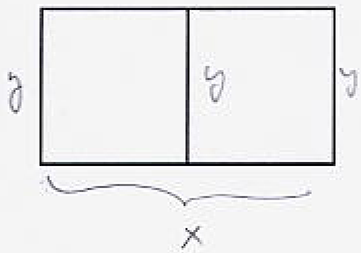
$\sqrt[3]{y} - 2 = 3x \quad | \div 3$

$x = \dfrac{\sqrt[3]{y}-2}{3}$

b) $(f \circ f^{-1})(y) = f(f^{-1}(y)) = f\left(\dfrac{\sqrt[3]{y}-2}{3}\right)$

$= \left(\cancel{3}\dfrac{\sqrt[3]{y}-2}{\cancel{3}} + 2\right)^3 = (\sqrt[3]{y})^3 = y$

7. (16pts) Farmer Tom has 5000 meters of fencing. He would like to enclose a rectangular area and divide it in half with a fence so that the area is the largest possible. Find the dimensions of the enclosure that will give the greatest area. What is the greatest area?



$2x+3y=5000$

$2x = 5000-3y$

$x = \frac{1}{2}(5000-3y)$

$A = xy = \frac{1}{2}(5000-3y)\cdot y$

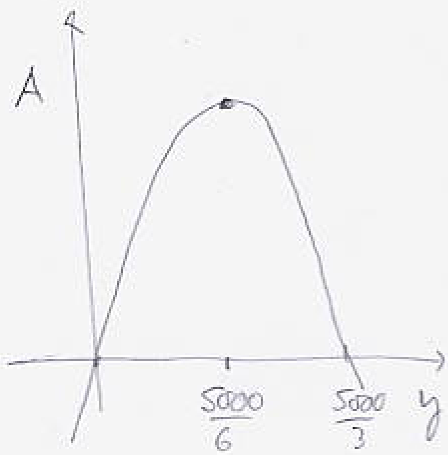
$= \frac{1}{2}(-3y^2+5000y) \leftarrow$ quadratic function

x-int

$y=0$

$5000-3y=0$

$y=\frac{5000}{3}$



Dimensions are: $y=\frac{5000}{6}$

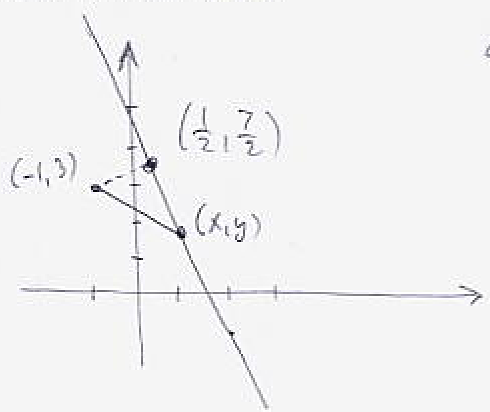
$x=\frac{1}{2}\left(5000-3\cdot\frac{5000}{6}\right)=\frac{1}{2}\cdot\frac{5000}{2}$

$=\frac{5000}{4}=1250$

Area $= \frac{\cancel{5000}^{2500}}{\cancel{6}_{3}} \cdot 1250 = \frac{3125000}{3}$

$= 1041{,}666.67\,m^2$

Bonus. (10pts) Find the point on the line $y = 5 - 3x$ that is closest to the point $(-1, 3)$. Draw a picture. Hints: Set up the expression for the distance d between a generic point (x, y) and the point $(-1, 3)$. Then express d only in terms of x, and minimize d^2 (you will need to simplify d^2).



$d=$ distance from $(-1,3)$ to (x,y)

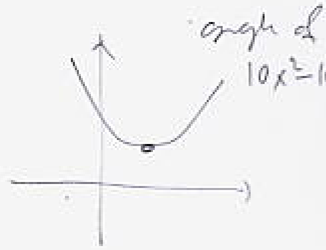
$d=\sqrt{(x-(-1))^2+(y-3)^2} \leftarrow$ point (x,y) is on the line so $y=5-3x$

$d^2=(x+1)^2+(5-3x-3)^2$

$d^2=(x+1)^2+(2-3x)^2$

$=x^2+2x+1+4-12x+9x^2$

$=10x^2-10x+5$



vertex: $x=-\frac{-10}{2\cdot 10}=\frac{1}{2}$

$y=5-3\cdot\frac{1}{2}=5-\frac{3}{2}=\frac{7}{2}$

Closest point is $(\frac{1}{2},\frac{7}{2})$ with distance $d^2=10\cdot\frac{1}{4}-10\cdot\frac{1}{2}+5$

$d^2=\frac{5}{2}$

$d=\sqrt{\frac{5}{2}}$