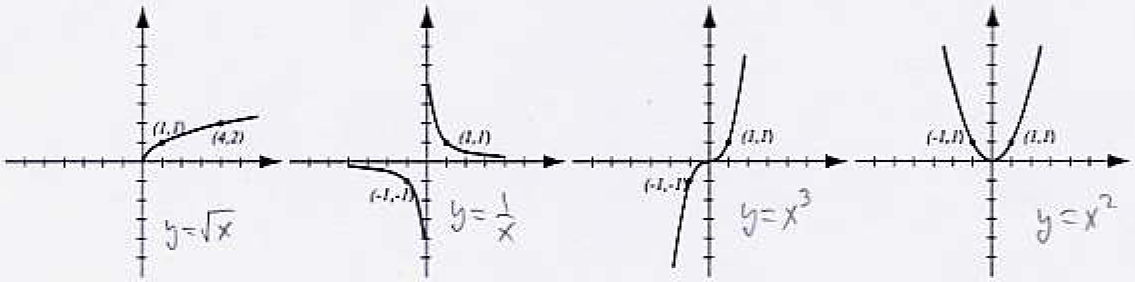
Spring '10/MAT 140/Exam 2 **Name:** Solution *Show all your work.*

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (11pts) Solve the inequalities and write the solution using interval notation:

$2x - 3 < 12 \quad |+3$

$2x < 15$

$x < \frac{15}{2}$

$\left(-\infty, \frac{15}{2}\right)$

$|x - 3| \geq 5$

distance from x to $3 \geq 5$

$(-\infty, -2] \cup [8, \infty)$

3. (8pts) Solve the equation.

$3x^2 - 5x = 2x + 6 \quad |-2x-6$

$3x^2 - 7x - 6 = 0$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4\cdot 3\cdot(-6)}}{2\cdot 3} = \frac{7 \pm \sqrt{49+72}}{6} = \frac{7 \pm \sqrt{121}}{6} = \frac{7 \pm 11}{6} = 3, -\frac{2}{3}$$

4. (10pts) Solve the equation.

$x^4 - 4x^2 - 21 = 0$ let $u = x^2$

$u^2 - 4u - 21 = 0$

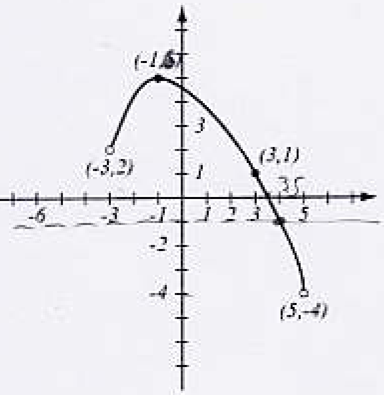
$(u-7)(u+3) = 0$

$u = 7$ or $u = -3$

$x^2 = 7$, $x^2 = -3$

$x = \pm\sqrt{7}$ $\quad x = \pm\sqrt{-3} = \pm\sqrt{3}\,i$

5. (10pts) Use the graph of the function f at right to answer the following questions.



a) What is the domain of f? $(-3, 5)$

b) What is the range of f? $(-4, 6]$

c) Find $f(-3)$ and $f(3)$. $f(-3)$ not defined. $f(3) = 1$

d) What are the solutions of the equation $f(x) = -1$? $x = 4$

e) Find all x for which $f(x) \geq 0$.

When graph is above x-axis

on $(-3, 3.5]$

6. (8pts) Find the equation of the line (in form $y = mx + b$) that is perpendicular to the line $3x - 4y = 7$ and passes through the point $(-1, 1)$.

$3x - 4y = 7$

$-4y = -3x + 7 \quad | \div -4$

$y = \frac{3}{4}x - \frac{7}{4}$

Slope $= \frac{3}{4}$

Slope of our line $= -\frac{4}{3}$

$y - 1 = -\frac{4}{3}(x - (-1))$

$y = -\frac{4}{3}x - \frac{4}{3} + 1$

$y = -\frac{4}{3}x - \frac{1}{3}$

7. (9pts) Below is an equation of a circle. Find the center and radius of the circle and draw the circle.

$x^2 + y^2 + 6x - 2y + 6 = 0$

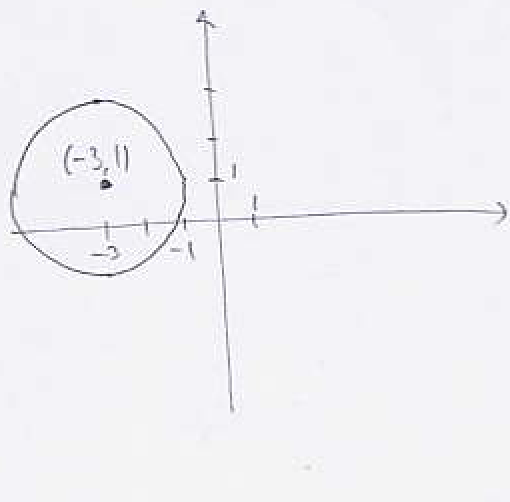
$x^2 + 6x + y^2 - 2y = -6 \quad | + 3^2 + 1^2$

$x^2 + 6x + 3^2 + y^2 - 2y + 1^2 = -6 + 10$

$(x+3)^2 + (y-1)^2 = 4$

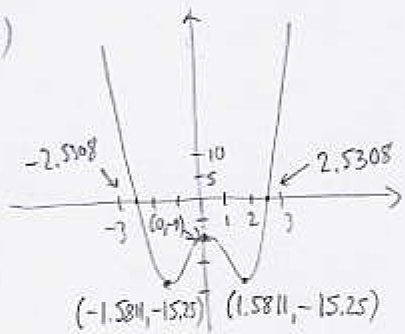
Center $= (-3, 1)$

radius $= \sqrt{4} = 2$



8. (22pts) Let $f(x) = x^4 - 5x^2 - 9$ (answer with 4 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.
b) Determine algebraically whether f is even, odd, or neither. Justify your answer further by examining the graph.
c) Find the x- and y-intercepts.
d) Find where f has a local minimum and maximum.
e) Find the intervals of increase and decrease.

a)



b) $f(-x) = (-x)^4 - 5(-x)^2 - 9$
$= x^4 - 5x^2 - 9$
$= f(x)$

f is even.
This is justified by the graph, which is symmetric about y-axis
(helps us find x-intercepts)

c) x-int: 2.5308
-2.5308
y-int: $f(0) = -9$

d) f has a local min at $x = 1.5811$ with value -15.25
at $x = -1.5811$ " -15.25

f has a local max at $x = 0$ with value -9

e) increasing on $(-1.5811, 0)$ and $(1.5811, \infty)$
decreasing on $(-\infty, -1.5811)$ and $(0, 1.5811)$

9. (6pts) Find the domain of the function $g(x) = \sqrt{3 - 4x}$.

Must have $3 - 4x \geq 0$
$3 \geq 4x \quad | \div 4$
$\frac{3}{4} \geq x$

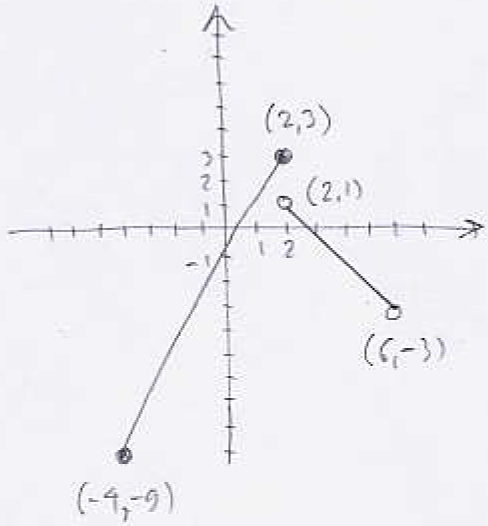
Domain is $(-\infty, \frac{3}{4}]$

10. (8pts) Sketch the graph of the piecewise-defined function:

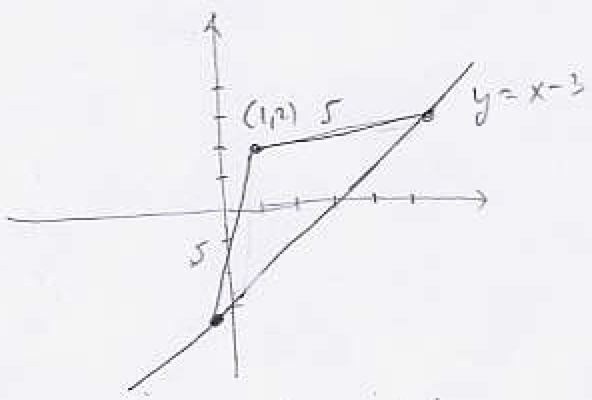
$$f(x) = \begin{cases} 2x - 1, & \text{if } -4 \le x \le 2 \\ -x + 3, & \text{if } 2 < x < 6. \end{cases}$$

x	2x−1
−4	−9
2	3

x	−x+3
2	1
6	−3



Bonus (10pts) Find all points on the line $y = x - 3$ whose distance to the point $(1, 2)$ equals 5. Draw a picture. *(Hint: Set up an equation involving the distance between a generic point* (x, y) *and the point* $(1, 2)$*. Then use the fact that the point* (x, y) *has to be on the line.)*



expect two solutions

$d((x,y),(1,2)) = 5$

$\sqrt{(x-1)^2 + (y-2)^2} = 5 \quad |^2$ must also have $y = x - 3$

$(x-1)^2 + (x-3-2)^2 = 25$

$(x-1)^2 + (x-5)^2 = 25$

$x^2 - 2x + 1 + x^2 - 10x + 25 = 25 \quad |-25$

$2x^2 - 12x + 1 = 0$

$x = \dfrac{12 \pm \sqrt{(-12)^2 - 4\cdot 2 \cdot 1}}{2 \cdot 2} = \dfrac{12 \pm \sqrt{144 - 8}}{4}$

$= \dfrac{12 \pm \sqrt{136}}{4} = \dfrac{12 \pm \sqrt{4 \cdot 34}}{4} = \dfrac{12 \pm 2\sqrt{34}}{4} = \dfrac{2(6 \pm \sqrt{34})}{4}$

$= \dfrac{6 \pm \sqrt{34}}{2} = 3 \pm \dfrac{\sqrt{34}}{2}$

The points are $\left(3 + \frac{\sqrt{34}}{2}, \frac{\sqrt{34}}{2}\right) \approx (5.9155, 2.9155)$

$\left(3 - \frac{\sqrt{34}}{2}, -\frac{\sqrt{34}}{2}\right) \approx (0.08452, -2.9155)$